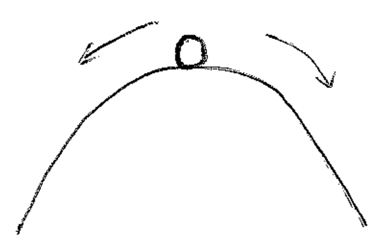
Last time: (03/02/12)

Phase Portraits for Nonlinear System

- A way to visualize all sol^{n}s (states) of a system
- Tells about qualitative features of a system's evolution.
- Equilibrium = System at rest;
  Closed orbit = Periodic Motion
  Limit Cycle = Special Closed Orbit.
- Stability (of equilibria or limit cycles)

Cartoon Picture of Stability:

stable



unstable

What if you don't like drawing pictures?
(and still wanted to know about stability)

Stability via linearization

03/02/12

$$\begin{cases} x' = f(x, y) \\ y' = g(x, y) \end{cases} \qquad x = x_e,\ y = y_e \text{ equilibrium}$$

Consider Jacobian

$$J = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix}$$

① If $J(x_e, y_e) = \begin{bmatrix} \frac{\partial f}{\partial x}(x_e, y_e) & \frac{\partial f}{\partial y}(x_e, y_e) \\ \frac{\partial g}{\partial x}(x_e, y_e) & \frac{\partial g}{\partial y}(x_e, y_e) \end{bmatrix}$

has all its eigenvalues negative or with negative real parts, then (x_e, y_e) is stable.

② If one of the eigenvalues of $J(x_e, y_e)$ is positive or has positive real part, then (x_e, y_e) is unstable.

③ Borderline cases can not be decided by eigenvalues of the Jacobian!