

Variation of Parameters

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Nonhomog System

$$(\star) \quad \boxed{\vec{x}' = A(t) \vec{x} + \vec{f}(t)}$$

← could depend on t !

Assume: homog. system

$$\vec{x}' = A(t) \vec{x}$$

can be solved by general solⁿ:

$$\vec{x}_n(t) = C_1 \vec{x}_1(t) + \dots + C_n \vec{x}_n(t)$$

w/ $\vec{x}_1(t), \dots, \vec{x}_n(t)$ linearly indep. homog. solⁿs.

Put $X(t) = [\vec{x}_1(t) \mid \dots \mid \vec{x}_n(t)]$ fundamental matrix

$$(\Rightarrow X'(t) = A(t) X(t))$$

$$\text{Then } \vec{x}_n(t) = X(t) \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$$

Variation of parameters means $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \rightsquigarrow \vec{v}(t)$

General solⁿ of nonhomog system

$$\boxed{\vec{x}(t) = X(t) \vec{v}(t)}$$

$$(\star) \Rightarrow \boxed{v'(t) = X^{-1}(t) \vec{f}(t)} \quad \text{or} \quad \vec{v}(t) = \int X^{-1}(t) \vec{f}(t) dt$$