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<u>matrix exponential</u> For an $n \times n$ matrix A,

$$e^{tA} = I + tA + \frac{t^2}{2!}A^2 + \frac{t^3}{3!}A^3 + \cdots$$

is again $n \times n$ matrix.

- $(e^{tA})' = A e^{tA}$ (e^{tA} is a funda. matrix)
- $e^{tA}\big|_{t=0} = I$
- $(e^{tA})^{-1} = e^{-tA}$.
- $e^{t(A+B)} \neq e^{tA} e^{tB}$ in general!

But when $\boxed{AB = BA} \Rightarrow e^{t(A+B)} = e^{tA} e^{tB}$.

- P invertible $\Rightarrow$ $\boxed{e^{t(PAP^{-1})} = P e^{tA} P^{-1}}$
- For $D = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$, $e^{tD} = \begin{bmatrix} e^{t\lambda_1} & & \\ & \ddots & \\ & & e^{t\lambda_n} \end{bmatrix}$

The last two help us compute e^{tA} by <u>diagonalization</u>.