Review: Ch4 (4.1 – 4.5), Ch5 (5.1 – 5.3).

Ch 4: 2^{nd} order linear diff. $eq^{\underline{n}}s$.

$$ay'' + by' + cy = f(t)$$

(1). homog. $f(t) = 0$.

char. $eq^{\underline{n}}$: $ar^2 + br + c = 0$

$\Rightarrow r_1, r_2$ roots.

a) distinct real roots $r_1 \neq r_2$

general $sol^{\underline{n}}$ $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

b). identical roots $r_1 = r_2$

$$y = c_1 e^{r_1 t} + c_2 t e^{r_2 t}$$

c). complex roots $r_1 = \alpha + i\beta$, $r_2 = \alpha - i\beta$

$$y = e^{\alpha t}(c_1 \cos\beta t + c_2 \sin\beta t)$$

(2). nonhomog. $f(t) \neq 0$.

general $sol^{\underline{n}}$ $y = y_h + y_p$

y_h solved as in 1.

y_p : method of undetermined coeff's.

What does $f(t)$ look like $\Longrightarrow$ What y_p should look like.

a) $f(t) = (a_n t^n + \cdots + a_1 t + a_0) e^{kt}$

$$\Rightarrow \quad y_p = \begin{cases} (A_n t^n + \cdots + A_1 t + A_0) e^{kt} & \text{if } k \neq r_1, k \neq r_2 \\ t(A_n t^n + \cdots + A_0) e^{kt} & \text{if } k = \text{one but not both of } r_1, r_2 \\ t^2 (A_n t^n + \cdots + A_0) e^{kt} & \text{if } k = r_1 = r_2. \end{cases}$$

b) $f(t) = e^{kt}(a \cos \omega t + b \sin \omega t)$

$$\Rightarrow \quad y_p = \begin{cases} e^{kt}(A \cos \omega t + B \sin \omega t) & \text{if } k + i\omega \neq r_1, r_2. \\ t e^{kt}(A \cos \omega t + B \sin \omega t) & \text{if } k + i\omega = \text{one of } r_1, r_2. \end{cases}$$

<u>Appl$^{\underline{n}}$</u>: Forced Oscillations

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos \omega_f t$$

① $b = 0$. resonance when $\omega_f = \sqrt{\frac{k}{m}}$

(when you need fudge factor "t")

② $b > 0$. $x = \underset{\substack{\uparrow \\ \text{transient sol}^{\underline{n}}}}{x_h} + \underset{\nwarrow \text{ steady-state sol}^{\underline{n}}}{x_p}$