Review:

Ch 5:

① Linear transf$^{\underline{n}}$s: $T: V \longrightarrow W$

s.t. $T(u+v) = T(u) + T(v)$

$T(ku) = k\,T(u)$ $\Rightarrow T(0) = 0$

for all $u, v \in V$, scalar k.

Linear transf$^{\underline{n}}$s $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ are always given by a matrix:

$$T(x) = Ax \quad \text{w/.} \quad A = \left[T(e_1) \mid \cdots \mid T(e_n) \right]$$

② Image and rank, kernel and nullity

$\operatorname{rank}(T) = \dim \operatorname{Im}(T)$

$\operatorname{nullity}(T) = \dim \ker(T)$.

For $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ given by $T(x) = Ax$,

$\operatorname{Im}(T) =$ span of column vectors of A

But to compute $\operatorname{rank}(T)$, need $A \xrightarrow{\text{EROS}} \text{RREF}$ to find linearly indep. column vectors.

Similarly, to compute $\ker(T)$ and $\operatorname{nullity}(T)$, use $A \xrightarrow{\text{EROS}} \text{RREF}$ to solve $Ax = 0$.

(3) Eigenvalues and eigenvectors

$T: V \longrightarrow V$ For $v \in V$, $\underline{v \neq 0}$

$$T(v) = \lambda v$$

eigenvalue (scalar)

eigenvector (vector)

For $T(x) = Ax$, $A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$, eigenvalues

are solved from characteristic eq$^{\underline{n}}$

$$\boxed{\det(A - \lambda I) = 0}$$

For a specific eigenvalues λ, the corresponding eigenvectors are found by solving

$$(A - \lambda I)v = 0$$

i.e. $A - \lambda I \xrightarrow{EROS} RREF$