Sol^n's HW 02-27-12

1. Matching: By sketching a few vectors along x-axis, y-axis, we see that the matches are C, A, B, D.

Stability: A: stable, since all trajectories near 0 stays close to 0 (circle around).

B: unstable, since some (in fact most of the) trajectories near 0 go away.

C: unstable, since all trajectories near 0 go away.

D: stable, since all trajectories near 0 spiral into it; hence stay close.

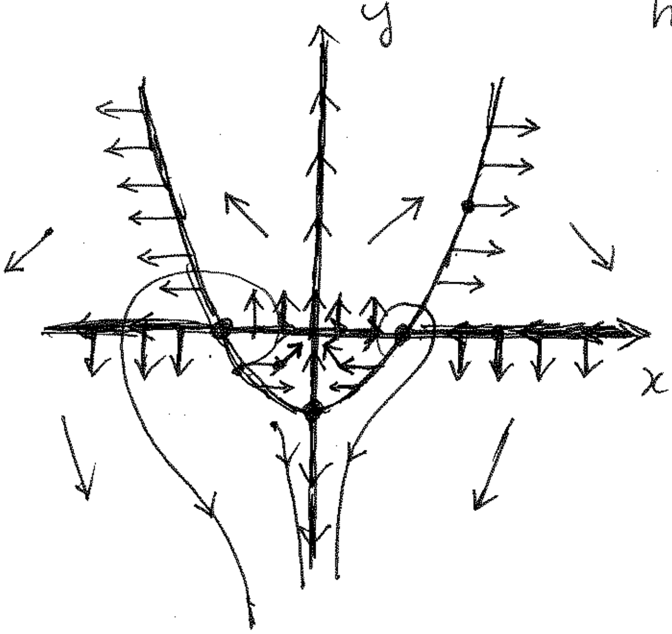
Phase Portraits from Nullclines

14. $\begin{cases} x' = xy \\ y' = y - x^2 + 1 \end{cases}$

v-nullcline: $xy = 0$ i.e. x-axis and y-axis

h-nullcline: $y - x^2 + 1 = 0$ i.e. $y = x^2 - 1$

equilibria = intersection of v-nullcline w/. h-nullcline $= (0,-1),\ (-1,0),\ (1,0)$



At $(\pm 2, 0)$, vector is $(0,-3)$

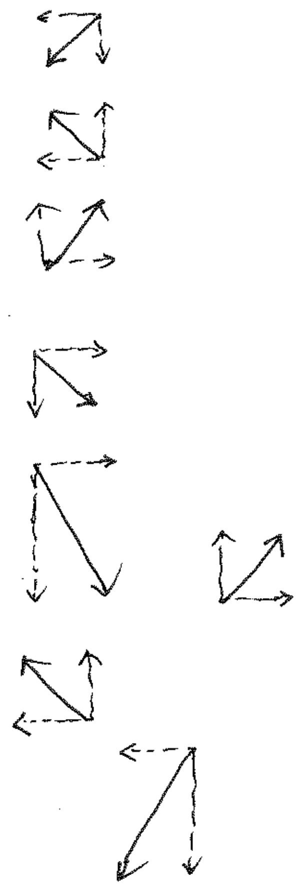
" $(0,0)$, " " $(0,1)$

" $(0,-2)$, " " $(0,-1)$

At $(\pm 2, 3)$, vector is $(\pm 6, 0)$; at $(\pm\frac{1}{2}, -\frac{3}{4})$, vector is $(\mp\frac{3}{8}, 0)$.

The nullclines divides the phase plane into 8 regions. We choose appropriate points to determine the directions of vector field in these regions:

At $(-2, 1)$, vector is $(-2, -2)$:
" $(-1, 1)$, " " $(-1, 1)$:
" $(1, 1)$, " " $(1, 1)$:
" $(2, 1)$, " " $(2, -2)$:
" $(-2, -1)$, " " $(2, -4)$:
" $(-\frac{1}{2}, -\frac{1}{2})$, " " $(\frac{1}{4}, \frac{1}{4})$:
" $(\frac{1}{2}, -\frac{1}{2})$, " " $(-\frac{1}{4}, \frac{1}{4})$:
" $(2, -1)$, " " $(-2, -4)$:



All equilibria are unstable.

17: $\begin{cases} x' = 1 - x^2 - y^2 \\ y' = x \end{cases}$

v-nullcline: $1 - x^2 - y^2 = 0$

h-nullcline: $x = 0$.

equilibria $(0, \pm 1)$.

$(0, 1)$ is stable;

$(0, -1)$ is unstable.