Sol^{n}s HW 03-02-12.

Original Equilibrium

(1) $\begin{cases} x' = -2x + 3y + xy \\ y' = -x + y - 2xy^2 \end{cases}$

To verify that $(0,0)$ is an equilibrium, we plug in $x = 0$, $y = 0$ to see that

$$\begin{cases} -2x + 3y + xy = -2\cdot 0 + 3\cdot 0 + 0\cdot 0 = 0 \\ -x + y - 2xy^2 = -0 + 0 - 2\cdot 0 \cdot 0^2 = 0 \end{cases}$$

$$J = \begin{bmatrix} -2+y & 3+x \\ -1-2y^2 & 1-4xy \end{bmatrix} \Rightarrow J(0,0) = \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix}$$

To find the eigenvalues of $J(0,0)$:

$$0 = \begin{vmatrix} -2-\lambda & 3 \\ -1 & 1-\lambda \end{vmatrix} = \lambda^2 + \lambda + 1 \Rightarrow \lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$\operatorname{Re} \lambda = -\frac{1}{2} < 0 \Rightarrow$ stable.

(6) $\begin{cases} x' = \sin y \\ y' = -\sin x + y \end{cases}$ $\quad \begin{cases} \sin 0 = 0 \\ -\sin 0 + 0 = 0 \end{cases} \Rightarrow (0,0)$ is equilibrium

$$J = \begin{bmatrix} 0 & \cos y \\ -\cos x & 1 \end{bmatrix} \quad J(0,0) = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \quad \lambda = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$\Rightarrow$ unstable.

Almost Linear

7 : $\begin{cases} x' = 1 - xy \\ y' = x - y^3 \end{cases}$ Equilibria : $\begin{cases} 1 - xy = 0 \\ x - y^3 = 0 \Rightarrow x = y^3 \end{cases} \Rightarrow 1 - y^4 = 0$

$\Rightarrow y = \pm 1$ (real sol^{n}s) and $x = \pm 1$.

i.e. $(1, 1)$, $(-1, -1)$.

$$J = \begin{bmatrix} -y & -x \\ 1 & -3y^2 \end{bmatrix} \qquad J(1,1) = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \qquad \lambda = -2, -2$$

$\Rightarrow (1, 1)$ is stable.

$$J(-1,-1) = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}, \quad \lambda = \underbrace{-1 + \sqrt{5}}_{>0}, \ -1 - \sqrt{5}$$

$\Rightarrow (-1, -1)$ is unstable.