Sol^n's HW 03-05-12

Original Equilibri

(2) $\begin{cases} x' = -y - x^3 \\ y' = x - y^3 \end{cases}$

Plug in $(0,0)$: $\begin{cases} -y - x^3 = 0 \\ x - y^3 = 0 \end{cases}$

$\Rightarrow$ equilibri

$$J = \begin{bmatrix} -3x^2 & -1 \\ 1 & -3y^2 \end{bmatrix} \qquad J(0,0) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \lambda = \pm i$$

can't tell !

(4) $\begin{cases} x' = y \\ y' = -\sin x - y \end{cases}$

$x = 0,\ y = 0 \Rightarrow \begin{cases} y = 0 \\ -\sin x - y = 0 \end{cases}$

$\Rightarrow$ equilibri

$$J = \begin{bmatrix} 0 & 1 \\ -\cos x & -1 \end{bmatrix} \qquad J(0,0) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \qquad \lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$\Rightarrow$ stable.

Almost Linear

(8) $\begin{cases} x' = x - 3y + 2xy \\ y' = 4x - 6y - xy \end{cases}$

Equilibri : $\begin{cases} x - 3y + 2xy = 0 \\ 4x - 6y - xy = 0 \end{cases}$

$\Downarrow$

$xy = 4x - 6y$

$\Rightarrow 0 = x - 3y + 2(4x - 6y) = 9x - 15y$

i.e. $x = \frac{5}{3} y \Rightarrow \frac{5}{3} y - 3y + \frac{10}{3} y^2 = 0$

$\Rightarrow y = 0$ or $y = \frac{2}{5} \Rightarrow x = 0$ or $x = \frac{2}{3}$.

$\Rightarrow$ equilibria $(0,0)$, $\left(\frac{2}{3}, \frac{2}{5}\right)$.

$$J = \begin{bmatrix} 1+2y & -3+2x \\ 4-y & -6-x \end{bmatrix} \qquad J(0,0) = \begin{bmatrix} 1 & -3 \\ 4 & -6 \end{bmatrix} \quad \lambda = -2, -3$$

$(0,0)$ is stable.

$$J\left(\tfrac{2}{3}, \tfrac{2}{5}\right) = \begin{bmatrix} 9/5 & -5/3 \\ 18/5 & -20/3 \end{bmatrix} \qquad \lambda = \frac{-73/15 \pm \sqrt{\left(\frac{73}{15}\right)^2 + 24}}{2}$$

$$\lambda_1 = \frac{-73/15 + \sqrt{\left(\frac{73}{15}\right)^2 + 24}}{2} > 0$$

$\Rightarrow \left(\frac{2}{3}, \frac{2}{5}\right)$ is unstable.

(9) $\begin{cases} x' = 4x - x^3 - xy^2 \\ y' = 4y - x^2y - y^3 \end{cases}$ Equilibri: $\begin{cases} 0 = 4x - x^3 - xy^2 \\ \quad = x(4 - x^2 - y^2) \\ 0 = 4y - x^2y - y^3 \\ \quad = y(4 - x^2 - y^2) \end{cases}$

$\Rightarrow$ $x = 0$ or $4 - x^2 - y^2 = 0$

And $y = 0$ or $4 - x^2 - y^2 = 0$

$\Rightarrow (0,0), (0, \pm 2), (\pm 2, 0)$.

$$J = \begin{bmatrix} 4 - 3x^2 - y^2 & -2xy \\ -2xy & 4 - x^2 - 3y^2 \end{bmatrix} \qquad J(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$\Rightarrow (0,0)$ unstable.

$$J(0, \pm 2) = \begin{bmatrix} 0 & 0 \\ 0 & -8 \end{bmatrix}$$

$(0, \pm 2)$ can not be determined.

$$J(\pm 2, 0) = \begin{bmatrix} -8 & 0 \\ 0 & 0 \end{bmatrix},$$

$(\pm 2, 0)$ can not be determined.

(11) Strong Spring $\ddot{x} + \dot{x} + x + x^3 = 0$

Set $y = \dot{x}$ $\Rightarrow$ $\begin{cases} x' = y \\ y' = -x - x^3 - y \end{cases}$

Equilibria: $\begin{cases} 0 = y \\ 0 = -x - x^3 - y \end{cases}$ $\Rightarrow$ $y=0$ and $x=0$ i.e. $(0,0)$.

$J = \begin{bmatrix} 0 & 1 \\ -1-3x^2 & -1 \end{bmatrix}$ $J(0,0) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ $\lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\Rightarrow$ $(0,0)$ is stable.

(12) Weak Spring $\ddot{x} + \dot{x} + x - x^3 = 0$

$\Rightarrow$ $\begin{cases} x' = y \\ y' = -x + x^3 - y \end{cases}$ equilibria $\begin{cases} 0 = y \\ 0 = -x + x^3 - y \end{cases}$

$\Rightarrow$ $y = 0$ and $x = 0$ or $x = \pm 1$

i.e. $(0,0)$, $(\pm 1, 0)$.

$J = \begin{bmatrix} 0 & 1 \\ -1+3x^2 & -1 \end{bmatrix}$ $J(0,0) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ $\lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$(0,0)$ is stable

$J(\pm 1, 0) = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ $\lambda = -2, 1$ $\Rightarrow$

$(\pm 1, 0)$ is unstable.