# Turbulence of Uniform Flow

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*Abstract*

We will discuss the turbulence problem and why is so important for our technology and the environment. Turbulence is one of the major problems holding our technology back and the relevance of turbulence to global warming and weather predictions will be discussed. We will also explain how all of this relates to analytical, numerical and experimental work currently being done at UCSB.

The news is that recent progress indicates that the turbulence problems can be solved in many if not all instances and we will explain the mathematics behind all of this without being too technical. Noise turns out to play a major role and stochastic PDEs.

The existence of stochastic processes will discussed, describing turbulent solutions of the full Navier-Stokes equation, driven by unidirectional flow, in dimensions one, two and three. The existence of these turbulent solutions can then be used to proof the existence of an invariant measure in dimensions one, two and three. The invariant measure characterizes the statistically stationary state of turbulence. They determine all the deterministic properties of turbulence and everything that can be computed and measured. In particular, the invariant measure determines the probability density of the turbulent solutions and this can be used to develop accurate sub-grid models in computations of turbulence, bypassing the problem that three-dimensional turbulence cannot be fully resolved with currently existing computer technology.