P.191 (2 points)
01 Incorrect
02 Correct (Prop 3.23 on p.128)
03 Incorrect (Thm. 4.4: Euclid’s V <-> Hilbert’s)
04 Incorrect (It does not guarantee existence)
05 Incorrect (That’s the converse!)
06 Correct (Because it just might NOT)
07 Incorrect
08 Correct (p.170 Thm 4.3 A (0))
09 Incorrect (p.115 Defn.)
10 Incorrect (p.164 Standard Construction)
14 Incorrect (The statement is actually false in hyperbolic geometry)
17 Correct (Construction on p.178)
20 Correct (Prop. 3.15 (a), p.125)
21 Correct

P.192

1a (1 point)
Converse: If two lines are intersected by a transversal in such a way that the two lines meet on one side of the transversal, then the sum of the degree measures of the two interior angles on that side of the transversal is less than 180°.

Let t be the transversal that intersect lines l and m (figure 1). Since the two lines meet on one side of t, let C be the point of intersection. Then \( \triangle ABC \) is a triangle and by Saccheri-Legendre theorem \((\angle A)^\circ + (\angle B)^\circ + (\angle C)^\circ \leq 180^\circ\). Since m and l are two distinct lines we have \((\angle C)^\circ \neq 0\), so we get \((\angle A)^\circ + (\angle B)^\circ < 180^\circ\) as desired. Alternative answer: By Cor. 2 of the Exterior Angle Theorem, any two interior angles of a triangle are less than 180°.

1c (1 point)
Let \(\Box ABCD\) (figure 2) be a Lambert quadrilateral, that is \(\angle BAC\), \(\angle ABD\) and \(\angle ACD\) are right angles. By Proposition 3.16 we can construct a perpendicular from C to \(\overrightarrow{BD}\) intersecting at E, so \(\angle BEC\) is a right angle (foot of the perpendicular). If \(D \neq E\), then CE and CD are two lines through C parallel to AB, contrary to Hilbert’s parallel postulate, thus \(D = E\). Then \(\angle BDC\) is a right angle, so \(\Box ABCD\) is a quadrilateral with four right angles and thus a rectangle.
Let $\square ABCD$ (figure 3) be a Saccheri quadrilateral, that is $\angle A$ and $\angle B$ are right angles and $AC \cong BD$. Let $E$ be the midpoint of $CD$, and by prop. 3.16 drop a perpendicular from $E$ to $BD$, intersecting at $F$. So $\angle BFE$ is a right angle. If $D \neq F$ then $\overrightarrow{EF}$ and $\overrightarrow{ED}$ are two lines through $E$ parallel to $AB$ contradicting Hilbert’s parallel postulate. So $F=G$, that is $\angle BDE$ is a right angle. Analogous argument gives us that $\angle ACE$ is a right angle. Thus all four angles of the Sacherri quadrilateral are right angles and so it is a rectangle.

By Prop. 4.11 any Hilbert plane that satisfies the parallel postulate is semi-Euclidean, then corollary 2 of the Uniformity Theorem gives us that a rectangle exist.

2 (2 points)
The assumption $(\angle C)^\circ = 180^\circ - (\angle A)^\circ - (\angle B)^\circ$ implies that the sum of the angles of any triangle is equal to $180^\circ$. But in neutral geometry by Saccheri-Legendre the sum of angles is less than or equal to $180^\circ$. The proof is valid in any semi-Euclidean Hilbert Plane, that is any Hilbert plane where every triangle has angle sum equal to $180^\circ$. Note: there exist semi-Euclidean plane that does not satisfy the parallel postulate, yet by Prop 4.11 any Hilbert Plane that satisfy the parallel postulate is semi-Euclidean.

3 (2 points)
(1) RAA hypothesis
(2) Proposition 3.13(a)
(3) Definition of $<$ for segments
(4) SAS
(5) Corresponding angles of congruent triangles are congruent
(6) CA5 (+transitivity of angle congruence). Recall that $\angle B \cong \angle E$ by hypothesis
(7) Exterior Angle Theorem (Theorem 4.2) and the definition of exterior angle. Note that we use the fact that $A * G * B$ from step (3) to get that $\angle AGC$ is exterior to $\triangle BCG$
(8) The assumption that $DE < AB$ from step (3) led to a contradiction
(9) symmetry - if we switch the roles of $A$ and $D$, of $B$ and $E$, and of $C$ and $F$, then the statement that we are trying to prove remains the same
(10) We have shown that both alternatives of step (2) lead to contradictions. Hence the RAA hypothesis (step (1)) must be false
(11) SAS
5 (2 points)
(1) Proposition 2.3
(2) CA4
(3) CA1
(4) Exercise 9, Chapter 3 - B is on the line $\overrightarrow{AB}$ and D and X are on a ray emanating from B. Hence, X and D are on the same side of $\overrightarrow{AB}$. By step (2), X and C are on opposite sides of $\overrightarrow{AB}$. Thus, by the corollary to BA4, D and C are on opposite sides of $\overrightarrow{AB}$.
(5) Definition of opposite side - the segment DC must intersect $\overrightarrow{AB}$.
(6) RAA
(7) If the first two cases (E = A, E = B) do not hold, then A, E, and B are three distinct points all lying on $\overrightarrow{AB}$. Hence, by BA3, precisely one of them is between the other two. The previous step assumes that E is not the “middle” one, which leaves the two cases stated.
(8) Alternate Interior Angle Theorem
(9) E is, by its definition, on $CD \in \overrightarrow{CD}$. If E were A, then, since $A \neq C$ from step 1, the two distinct points A = E and C would both lie on $\overrightarrow{CD}$ and also on $\overrightarrow{AC}$. By the uniqueness part of IA1, we would therefore have $\overrightarrow{AC} = \overrightarrow{CD}$. In particular, this would mean that D lies on $\overrightarrow{AC}$. This contradicts the previous step (since lines $\overrightarrow{AC}$ and $\overrightarrow{BD}$ would intersect at D). An entirely symmetrical argument shows that $E \neq B$.
(10) Proof by cases. According to Step (7), there are four possible cases in our RAA argument. The previous step shows that the first two lead immediately to contradictions. We’ll now show, in steps (11) through (13), that the third case also leads to a contradiction.
(11) Pasch’s Theorem
(12) This step is a good example of why I don’t like the author’s “numbered statement” approach. Here, he makes a statement that has multiple reasons. If $\overrightarrow{AC}$ intersects ED, then $\overrightarrow{AC}$ and $\overrightarrow{CD}$ must be the same, since they share the distinct points C and some point of the segment ED. (Note that $C \neq D$, which comes from the definition of opposite side, prevents C from being a point of ED, thus assuring that C is distinct from the “some point of ED.”) Thus, since D is on $\overrightarrow{CD}$, it must also be on $\overrightarrow{AC}$, which means that $\overrightarrow{AC}$ and $\overrightarrow{BD}$ intersect at D, a clear impossibility in light of step (8). If, on the other hand, $\overrightarrow{AC}$ intersects BD we get the desired contradiction directly from step (8).
(13) The assumption of step (10) has led to a contradiction.
(14) Symmetry
(15) All possible cases of the RAA assumption of step (6) have led to contradictions.
(16) Vertical angles are congruent. Note that we needed $A \neq E \neq B$ to get vertical angles - this assures that $\overrightarrow{EA}$ is opposite to $\overrightarrow{EB}$ by Proposition 3.6.
(17) SAA
(18) Corresponding sides of congruent triangles are congruent; definition of midpoint.