MATH 116

Midterm

50 points

Instructions: Read each question carefully. Each problem is worth **10 points**. Write your solutions on the colored paper. Do not turn in scratch work. Good luck.

1. There are \( n \) married couples at a party. Each person shakes hands with every person other than his or her spouse. Find the total number of handshakes.

2. Let \( S \) be the set \( S = \{1, 2, \ldots, 10\} \). Find the number of subsets of \( S \) that contain the number 1, 3, or 7.

3. We are to seat five boys, five girls, and one parent in at a circular table with eleven seats. In how many ways can this be done if no boy is to sit next to a boy and no girl is to sit next to a girl?

4. How many sets of three integers between 1 and 20 are possible if no two consecutive integers are to be in a set?

5. Let \( n \) be a positive integer, and let \( S \) be an \( n \)-element set. If \( A \) is a subset of \( S \), denote by \( o(A) \) the number of elements in \( A \) (in particular, \( o(S) = n \)). Say that \( A \) is odd if \( o(A) \) is odd, and \( A \) is even if \( o(A) \) is even (the integer 0 is considered even). Prove that the number of odd subsets of \( S \) equals the number of its even subsets. Hint: Consider the cases \( n \) odd and \( n \) even separately.