1. \( X \) is **not** Hausdorff. Hausdorff condition fails on \( x_1 = 0^+ \) and \( x_2 = 0^- \).

2. We showed in HW that \( A \subseteq X \) is open iff \( \exists A \subseteq X - A \), and \( A \subseteq X \) is closed iff \( \exists A \subseteq A \).

So \( A \) both open \& closed iff \( \exists A \subseteq X - A \) and \( \exists A \subseteq A \).

This can happen if and only if \( \exists A = \emptyset \).

(b) First, show \( \overline{A} \subseteq A \cup \exists A \).

Let \( x \in \overline{A} \).

We know \( \overline{A} = A \cup A' \).

So \( x \in \overline{A} \) implies that \( x \in A \) or \( x \in A' \).

If \( x \in A \), we are done.

Now suppose \( x \in A' \setminus A \).
We know that \( 2A = \overline{A} \cap X-A \).

\( x \in X-A \) implies that \( x \in X-A \).

Since \( X-A \supseteq X-A \),

so \( x \in \overline{A} \cap X-A \) i.e. \( x \in 2A \).

Now show \( A \cup 2A \subseteq \overline{A} \).

Assume \( x \in A \cup 2A \).

If \( x \in A \), we are done since \( \overline{A} \supseteq A \).

Suppose \( x \notin A \) and \( x \in 2A \).

But \( 2A = \overline{A} \cap X-A \), so \( x \in \overline{A} \).

3. Let \( U \in \overline{A}_x \), i.e. \( U \) is a subset of \( A \) which is open in \( X \). Know that

\( U = U \cap Y \), so \( U \) is also open in \( Y \).

\( \therefore U \subseteq \overline{A}_y \).
4. Recall that for $C \subseteq X$, a subset $C$ of $X$, $x \in \overline{C}$ if and only if every nbhd of $x$ intersects $C$.

First show $\overline{A \times B} \subseteq \overline{A} \times \overline{B}$.

Let $(x, y) \in \overline{A \times B}$, wts: $(x, y) \in \overline{A} \times \overline{B}$

That is, wts $x \in \overline{A}$ & $y \in \overline{B}$.

Let $U$ be a nbhd of $x$ & $V$ be a nbhd of $y$. Then $U \times V$ is a nbhd of $(x, y)$. By hypothesis, $U \times V$ intersects $A \times B$. Say $(c, d) \in (U \times V) \cap (A \times B)$.

But $(U \times V) \cap (A \times B) = (U \cap A) \times (V \cap B)$.

So $c \in U \cap A$ and $d \in V \cap B$.

Thus $x \in \overline{A} \times \overline{B}$. 
Now show $\overline{A \times B} \subseteq \overline{A \times B}$.

Let $(x, y) \in \overline{A \times B}$, i.e. $x \in \overline{A}$ & $y \in \overline{B}$.

Wts: $(x, y) \in \overline{A \times B}$.

Let $W$ be a nbhd of $(x, y)$. By defn of product topology on $X \times Y$, there is a basis element $(U \times V)$ containing $(x, y)$ sitting inside of $W$, where $U$ is open in $X$ & $V$ is open in $Y$.

So $U$ is a nbhd of $x$ & $V$ is a nbhd of $y$. Since $x \in \overline{A}$, $y \in \overline{B}$, we know that $A \cap U$ contains some point $c$ & $B \cap V$ contains some point $d$.

So $(c, d) \in (A \cap U) \times (B \cap V) = (A \times B) \cap (U \times V)$.

Thus $(c, d) \in (A \times B) \cap W$, since $U \times V \subseteq W$. [\qed]
5. a. Let $U \subseteq Y$ be open.

Then $f^{-1}(U)$ is some subset of $X$. Since $X$ has discrete topology, $f^{-1}(U)$ must be open in $X$.

\[ \therefore f \text{ is continuous} \]

(b) Let $Y$ be the topological space $(X, J_d)$ where $J_d$ is the discrete topology on $X$.

Let $f: X \to Y$ be the identity map.

Let $U \subseteq X$ be some subset of $X$ which is not open in $X$. We know there is such a thing since $X$ is not discrete.

We know that $U \subseteq Y$ is open, since $Y$ is discrete. But $f^{-1}(U) = U$ is not open in $X$. Thus, $f$ is not continuous.
Let $X$ be the topological space that is $(Y, J_i)$ when $J_i$ is the indiscrete topology.

Let $f: X \rightarrow Y$ be the identity map.

Since $Y$ is not indiscrete, we can find a set $U \subseteq Y$ which is open but $U \neq \emptyset$ and $U \neq Y$.

However, $f^{-1}(U) = U$ is not open in $X$ because the only sets open in $X$ are $\emptyset$ & $X$. 

\[\square\]