1. Metric spaces are Hausdorff, and compact subspaces of Hausdorff spaces are closed.

2. Let \( J \) be a topology on \( A \) and suppose that \( p: X \rightarrow A \) is continuous.

\[ \text{wts: } J \subseteq J_0, \text{ where } J_0 \text{ is the quotient topology on } A \text{ induced by } p. \]

Let \( U \in J \). Since \( p \) is continuous, we know that \( p^{-1}(U) \) is open in \( X \). Thus, by defn of \( J_0 \), \( U \in J_0 \).
3. Suppose, on the contrary, that $A \not\subseteq B$.

Then $C = A \cap B$ is a proper, nonempty subset of $A$.

Also notice that $C$ is both open & closed in $A$ since $B$ is both open & closed in $X$.

$\longrightarrow$ $A$ is connected

$A \subseteq C \subseteq B$
4. Let $A$ be an open cover of $(X, J)$. Since $J \subseteq J'$, every element of $J$ is also open in $(X, J')$. Thus $A$ is also an open cover of $(X, J')$. By compactness of $(X, J')$ we can reduce $A$ to a finite subcover of $(X, J')$. This is also a finite subcover of $(X, J)$. 

5. Suppose $x, y$ are two points in $A$. Then there is a path from $x$ to $y$ since $A$ is path-connected. Similarly, we can find a path from $x$ to $y$ for $x, y$ in $B$. Now suppose $x \in A$ and $y \in B$. Since $A \cap B \neq \emptyset$, choose $z \in A \cap B$. There is a path from $x$ to $z$ (since $A$ path-connected) and a path from $z$ to $y$ (since $B$ path-connected). The composition of
these two paths is a path from $x$ to $y$. 