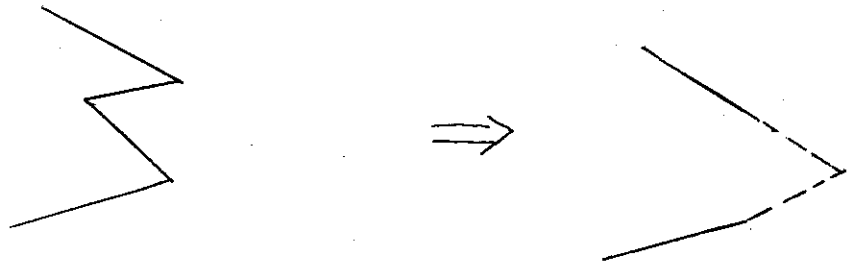
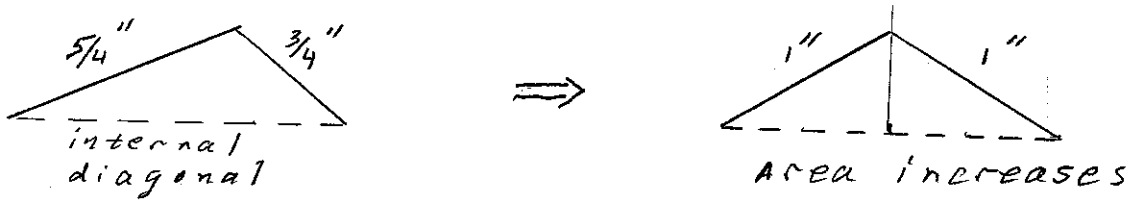
Of all polygons with <u>given</u> perimeter and N sides, the regular polygon has largest area.

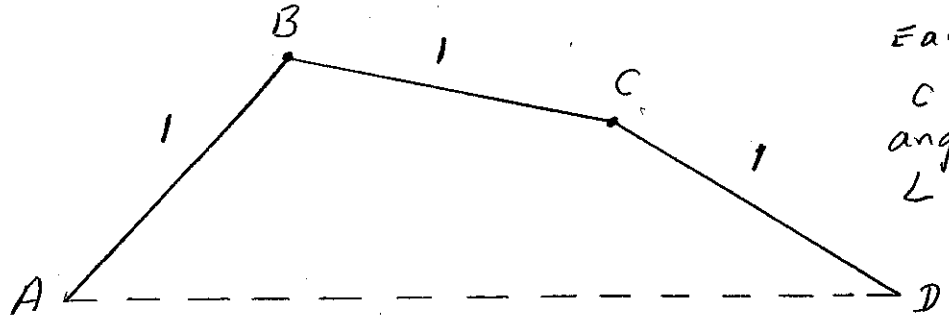
(I) A maximal area polygon is convex:



(II) A maximal polygon has equal sides.



(III) A maximal polygon has equal angles.



Each side is 1. Consecutive angles $\angle ABC$, $\angle BCD$, differ.

Moving B and C to make the angles the same increases the area.

EqualAngles

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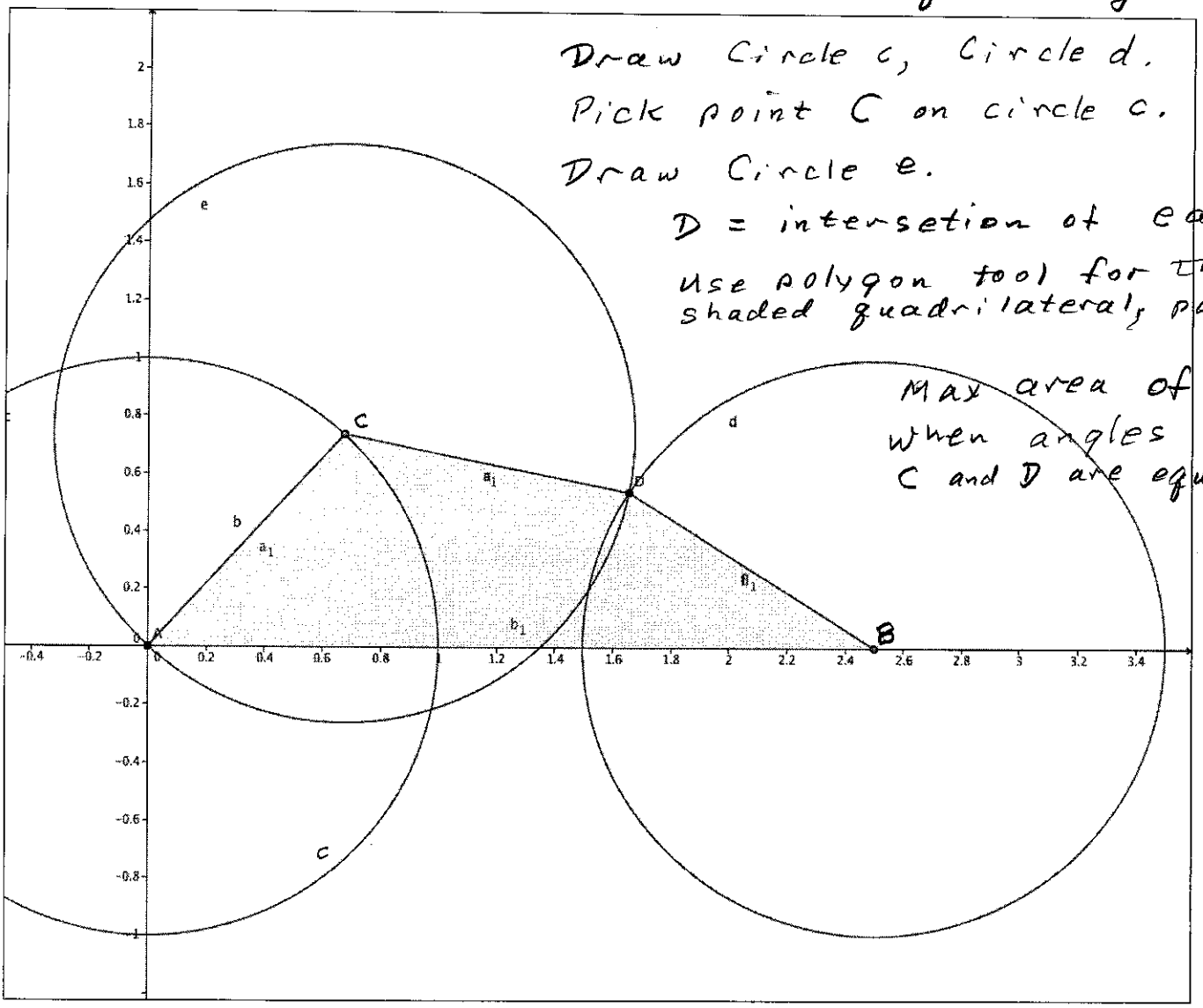
$\overline{AB}$: diagonal across three sides.
All sides are equal, length 1.



No.	Name	Definition	Algebra
1	Point A	intersection point of xAxis, yAxis	A = (0, 0)
2	Circle c	Circle with center A and Radius 1	c: $x^2 + y^2 = 1$
3	Point B	Point on xAxis	B = (2.499, 0)
4	Circle d	Circle with center B and Radius 1	d: $(x - 2.499)^2 + y^2 = 1$
5	Point C	Point on c	C = (0.676, 0.737)
6	Circle e	Circle with center C and Radius 1	e: $(x - 0.676)^2 + (y - 0.737)^2 = 1$
7	Point D	intersection point of d, e	D = (1.655, 0.538)
8	Segment a	Segment[C, D]	a = 1
9	Segment b	Segment[C, A]	b = 1
10	Segment f	Segment[D, B]	f = 1
11	Quadrilateral poly1	Polygon A, C, D, B	poly1 = 1.1
11	Segment a_1	Segment[A, C] of Quadrilateral poly1	a_1 = 1
11	Segment c_1	Segment[C, D] of Quadrilateral poly1	c_1 = 1
11	Segment d_1	Segment[D, B] of Quadrilateral poly1	d_1 = 1
11	Segment b_1	Segment[B, A] of Quadrilateral poly1	b_1 = 2.499

← Move C to change drawing.

← Max area is when C is positioned so angles at C, D are the same.

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EqualSides

J. Doner

$\overline{AB}$: a diagonal of the given polygon, cutting across two sides.

$\overline{CD}$: line representing length of ~~the two~~ sides.

Here, $\overline{AB} = 2$ (convenient)

$\overline{CD} = 2.7$ (arbitrary, but ≥ 2).

Pick E on $\overline{CD}$, to determine the two sides of length c, d.

Draw Circle e.

Draw Circle f.

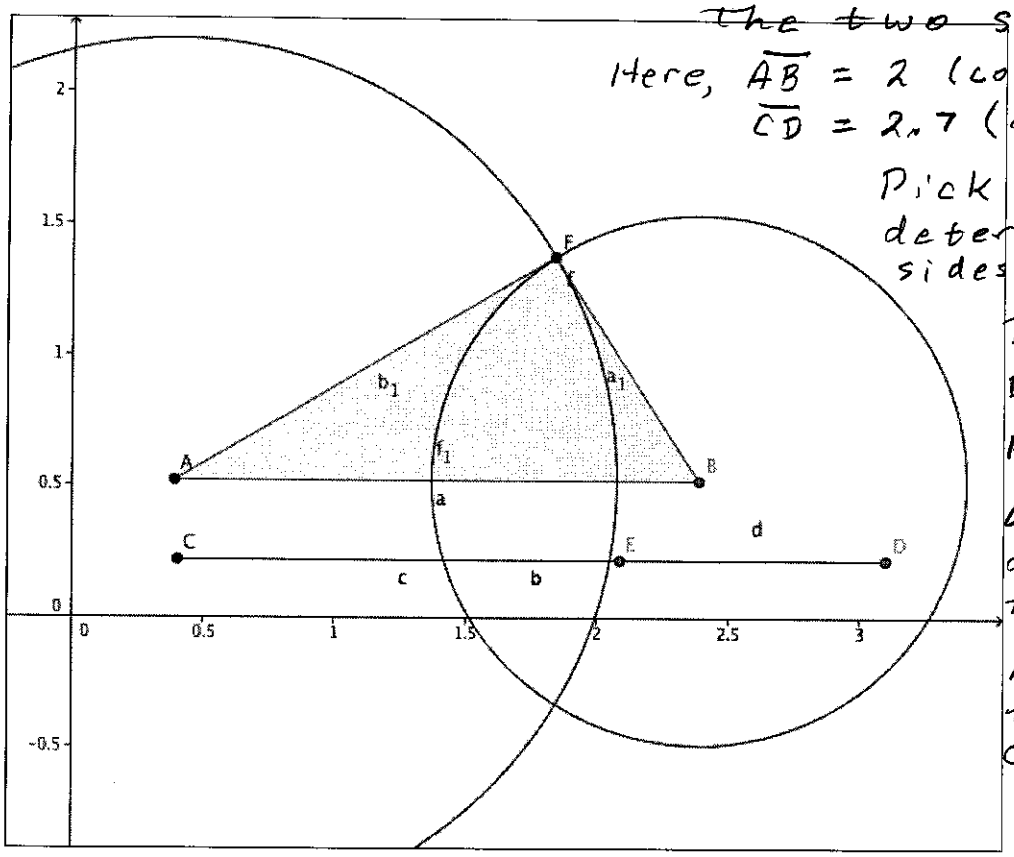
F is their intersection.

Use polygon tool to define the shaded triangle, corners A, F, B

Area of poly1 (the triangle) is automatically computed.

Move E to change c, d and hence, F.

Max area when c = d



No.	Name	Definition	Algebra
1	Point A		A = (0.39037, 0.52148)
2	Point B	Point on Circle[A, 2]	B = (2.39037, 0.52148)
3	Segment a	Segment[A, B]	a = 2
4	Point C		C = (0.40222, 0.21926)
5	Point D	Point on Circle[C, 2.7]	D = (3.10222, 0.21926)
6	Segment b	Segment[C, D]	b = 2.7
7	Point E	Point on b	E = (2.08519, 0.21926)
8	Segment c	Segment[C, E]	c = 1.68296
9	Segment d	Segment[D, E]	d = 1.01704
10	Circle e	Circle with center A and Radius c	e: $(x - 0.39037)^2 + (y - 0.52148)^2 = 2.83236$
11	Circle f	Circle with center B and Radius d	f: $(x - 2.39037)^2 + (y - 0.52148)^2 = 1.03436$
12	Point F	intersection point of e, f	F = (1.83987, 1.37665)
13	Triangle poly1	Polygon A, F, B	poly1 = 0.85517
13	Segment b_1	Segment[A, F] of Triangle poly1	b_1 = 1.68296
13	Segment a_1	Segment[F, B] of Triangle poly1	a_1 = 1.01704
13	Segment f_1	Segment[B, A] of Triangle poly1	f_1 = 2

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