Geometry

\[ M \geq |Z| \]
\[ X : CY_3, \ y \in H_*(X), \ \text{where are there volume-minimising cycles in class } y \]

\[ F \text{calibration on } X \]
\[ \omega \in \Omega^*(X), \ \text{cl} \omega = 0 \]

\[ \int \omega \leq \text{vol}(Y) \]
\[ y \in C_y \]
\[ \uparrow \]
\[ \text{Mass of BPS charge of the cycle} \]

\[ R. \; Hanany \]
\[ B. \; Kawamura, \sim 1980 \]

\[ \omega = \text{Kähler form} \]
\[ \Omega = \text{holom n-form on CY n-fold} \]

Physics

\[ 4D \quad N = 2 \; \text{SUGRA} \]
\[ \text{(type IIA, IIB on CY_3)} \]
\[ 4D \quad N = 2 \; \text{Susy} \]
\[ \text{(... on non-cpt. CY_3)} \]

\[ \text{Superalgebra: Poincaré reps of superalgebra:} \]
\[ \text{BPS massive reps} \]
\[ M \geq |Z| \; \text{BPS charge saturated by BPS states} \]

\[ \text{given a possible \#charge of possible massive states how many BPS states are there?} \]

\[ \sum_{\alpha \in \text{Lattice}} n_{\alpha} g_{\alpha} \quad \text{spec. for BPS spectrum} \]
3-cycles on CY calibrated by holom 3-form \( \Omega \) special h. a. 3-cycles” \{ no tech. available \} (D3-brane wrapped, in II B) \( \omega \) \( k \)-cycles on CY calibrated by \( \omega \) good \( \omega \) \( k \) complex subvarieties” \{ no tech. available \} (D2k-brane, in II A)

Seiberg-Witten: \( N = 2 \) susy QFT \( SU(2) \), no matter \( \mathbb{C} \) cplx. charge \( U \in \mathbb{C} \) (single parameter) \( Z = Z(\omega) \) \( \text{holomorphic} \) (weakly coupled \( \leftrightarrow \) perturbative techn.) \( \text{strongly coupled} \)

\[ \times \times \]

(not single-valued)

spectrum can be calculated at large \( u \)

\[ M(\eta, n_\rho) \neq 0 \]

\[ \leftrightarrow (\eta, n_\rho) = (0, \pm 1) \]

or \( (\pm 1, k) \)

Monodromy \( \text{in } (n_\rho, n_\mu) \)

\( \Omega(\Omega) \subset SL(2, \mathbb{R}) \)
- At the 2 bad pts some BPS state becomes massless.

- Spectrum can change as you cross at wall
  (BPS spectrum jumping
  $\Rightarrow$ jumping wall / jumping line
  (wall of marginal stability)

\[ u \text{-plane} \leftrightarrow \text{cplx Kähler class of CY} \]

\[ \omega + iB \]
Stability in moduli problems in A.G.

\[ \{ \text{alg. geom. objects labelled by } z_1, \ldots, z_k \} / \text{cyl. sym. grp} \]

e.g.
\[ \left( \frac{z_1, z_2}{\mathbb{C}^*} \right) \cong (z_1, z_2) \sim (lz_1, l^2z_2) \]

\((0, 0) \text{ lies in closure of every orbit.}\)

\[ \left\{ (z_1, z_2) \mid (z_1, z_2) \neq (0, 0) \right\} / \mathbb{C}^* \cong \mathbb{CP}^1 \]

Vector-bundles on Riem. Surf.

\[ W \subseteq V \quad \text{bad pts} \quad \frac{\deg(w)}{\operatorname{rk}(w)} > \frac{\deg(v)}{\operatorname{rk}(v)} \]

**Mumford-Takemoto Stability**

\[ W \subseteq V \quad X^d \quad \text{proj alg. variety} \]

\[ H = \text{ample divisor} \]

\[ \mu_H(W) = \frac{c_1(W) \cdot H^{d-1}}{\operatorname{rk}(w)} \]

\[ \text{stable } \forall W \subseteq V, \quad \operatorname{rk} W < \operatorname{rk} V \]

we have \( \mu_H(W) < \mu_H(V) \)
semistable \quad \text{same w/} \quad \mu_H(w) \leq \mu_H(v)

\Rightarrow \text{you have moduli spaces (given H)}

\text{Ms. stab}_H \supseteq \text{Ms. stab}_{t H}

\text{cnt.}

\text{as you vary } H \quad \text{(studied by Thaddeus)}

H^2(X) \cong \mathcal{K}(X) = \text{Kähler cone}

\bigcup \left[ H \right]

\text{\checkmark chambers w/ walls}

\text{BPS jumping in alg. geom}

\text{formalized by Bridgeland}

\text{abstract stability conditions}
Nagao - Nakajima
non-projective CY

- countable number of lines
- accumulate towards $y = -x$
- Kontsevich - Soibelman
- Gaiotto - Moore - Neitzke