2-Manifolds, 3-Manifolds & SUSY gauge theories

(D. Morrison)

Starting point in physics: \( A_{n-1} \leftrightarrow SU(N) \)

Six-dimensional: \( N \) coincident M5-branes in M-theory in \( M^{10,1} \)

(\( M2 \)-branes)

Indirect study: \( M^{k,3} \times X^{5-d-k} \)

\( 5-k=1 \) \( M^{4,3} \times S^1 \) map to gauge field theory \( G = SU(N) \)

Circumference \( \leftrightarrow \) coupling of gauge theory

\( M^{3,1} \times \Sigma \)

\( \Sigma \) Riemann surface (punctured allowed) \( \Rightarrow \) Complicated 4d physical theory

Wilson line operators

\( M2 \subset M5 \)

\( M5 \) on \( \Sigma \times R^{3,1} \)

\( M2 \) on \( Y \times R^{1,1} \)

For appropriate metric on \( \Sigma \)

\( SU(2) \)

\( SU(2) \) gauge theories

\( g \) -3 \( SU(2) \)'s (if no punctures, complex)
4D Theory: Witten- 't Hooft line operators

\[ q = \text{electric charge} \]
\[ p = \text{magnetic charge} \]

Fenchel - Nielsen

Hypothesis: \( \gamma \) is non-self-intersecting

Data: \# Crossing, \# winding near each separating geodesic

Dehn - Thurston Thm

\[ \pi, \text{ even} \]
\[ \pi, \pi, \pi > 0 \]

Lemma

\[ H: \text{homotopy class of curves} \]
\[ \# \text{of boundaries} \]
3-Manifolds

"quaternions", i.e., (hyperbolic) metrics with cusp in a knot/link in $\mathbb{R}^3$

decomposes 3-manifold into "ideal tetrahedra"

Cross ratios

3+1 dim

2+1 dim' l object

Coupling between 4D theory
3D boundary theory

Domain wall
other domain

Domain in $\mathbb{R}^4$