A Landscape of Field Theories

Travis Maxfield

Enrico Fermi Institute, University of Chicago

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Despite the recent proliferation of supersymmetric field theories on or associated to curved manifolds, we argue that a vast, uncharted territory remains.

String theory fluxes are responsible for this generalization.

Excursions have yielded many insights...

- Topological field theory, four manifold invariants

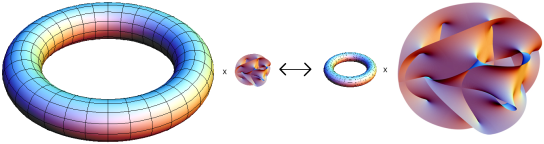
  [Witten, ...]

- "Geometric engineering" of interesting and practical field theories as limits of string theory in various backgrounds

  [Bershadsky et. al., ...]

- Indices, partition functions, dualities from compactification on spheres, etc.

  [Pestun, ...]



An example: $6D$, $(2,0)$ maximal supersymmetry

- Low-energy limit of single M5-brane
- Chiral two form: $dB_2 = H_3 = *H_3$
- Five scalars ϕ^I, $SO(5)$ global symmetry
- Four chiral fermions, Θ

On $K3 \leftrightarrow$ heterotic on T^3

Reduce H_3 on harmonic 2-forms of $K3$

$$H_3 = \sum_{i=1}^{19} \partial_- X^i \, \omega_i^- + \sum_{i=1}^{3} \partial_+ X^i \, \omega_i^+$$

- 19 ASD forms
- 3 SD forms
- X^i describe the moduli space

$$\frac{O(3,19)}{O(3) \times O(19) \times O(3,19;\mathbb{Z})}$$

- Same as heterotic string on T^3, a free theory

How to preserve supersymmetry on a curved manifold?

Three degrees of generalization,

- Killing spinor
- Topological twist
- Off-shell background supergravity

Let's examine the pertinent details...

Minimally coupling a field theory to gravity requires

$$\nabla_\mu \epsilon := \partial_\mu \epsilon + \omega_\mu \epsilon = 0.$$

- Quite restrictive
- Minkowski and tori are prime examples
- Generally Calabi-Yau and other spaces of special holonomy

Include background global symmetries, if they exist

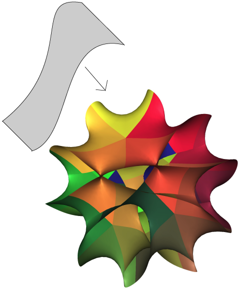
$$\partial_\mu \epsilon + (\omega_\mu - A_\mu)\epsilon = 0.$$

- Choosing $A_\mu = \omega_\mu$, Killing spinors are constant:

$$\partial_\mu \epsilon = 0.$$

- Topological twisting of [Witten]

- In string theory, branes wrapping cycles of SUSY backgrounds.
  [Bershadsky, Sadov, Vafa].



Make more concrete:

- In flat, D-dimensional space, p-brane breaks the Lorentz group:

$$SO(D-1,1) \longrightarrow SO(p,1) \times SO(D-p-1).$$

- Worldvolume Lorentz
- Global R-symmetry
- D-dimensional background metric couples to both symmetry currents.
- Supersymmetric worldvolumes have these cancel on the brane

Is an on-shell approach. Backgrounds solve D-dimensional SUGRA EOM.

Off-shell approach systematized by [Festuccia-Seiberg, ...]

- Couple theory to background, off-shell, $(p+1)$-dimensional SUGRA
- SUGRA auxiliary fields introduce background manifold
- Supersymmetry requires

$$\nabla_\mu \epsilon - A_\mu \epsilon = V_\mu \epsilon + V^\nu \sigma_{\mu\nu} \epsilon$$

- Solutions include novel manifolds for field theories

[Dumitrescu-Seiberg, ...]

$$S^n, \quad \mathbb{R}^k \times S^n, \quad S^1 \times S^n, \ \ldots$$

For example: 4 supercharges $\Rightarrow \quad \mathbb{R} \times \mathcal{M}_3$

- To preserve 4 supercharges,

$$\nabla_\mu V_\nu = 0, \quad R_{\mu\nu} = -2\left(V_\mu V_\nu - g_{\mu\nu} V_\rho V^\rho\right)$$

- Constant curvature with parallel vector $\Rightarrow$ locally $\mathbb{R} \times \mathcal{M}_3$

$$\mathcal{M}_3 = S^3, \quad \mathbb{R}^3, \quad H^3$$

- For S^3, choose $V_\mu = \frac{i}{r}$
- r is radius of S^3.

Summary:

- Preserve supersymmetry by coupling to background fields
- Can be on-shell $\leftarrow$ string backgrounds and brane physics
- Or off-shell $\leftarrow$ SUGRA auxiliary fields

Generalize the on-shell method by including the “landscape” of supersymmetric flux vacua

Hints of a relationship to off-shell method

What is this “landscape”? A specific case:

- M-theory on $\mathbb{R}^{1,2} \times \mathcal{M}_8$ with G_4 flux
- Supersymmetry requires

$$\delta\psi_M = \nabla_M \epsilon + \frac{1}{12}\left(\Gamma_M \not{G}_4 - 3(\not{G}_4)_M\right)\epsilon = 0.$$

This condition leads to study of “G-structures”.

One class of solutions has $\mathcal{M}_8 = CY_4$

[Becker-Becker]

- Metric

$$ds^2 = \Delta(y)^{-1}\eta_{\mu\nu}dx^\mu dx^\nu + \Delta(y)^{\frac{1}{2}}ds^2_{\mathcal{M}_8}(y).$$

- Flux EOM requires

$$G_4 \in H^{(4,0)} \oplus H^{(2,2)} \oplus H^{(0,4)}$$

- These don't generically preserve SUSY

- In low-energy SUGRA, flux-induced superpotential

[Gukov-Vafa-Witten]

$$W = \int_{\mathcal{M}_8} \Omega_4 \wedge G_4$$

On a general CY_4 (holonomy $= SU(4)$), fluxes can preserve either

- $3D$ $N = 2$ supersymmetry, i.e. 4 real supercharges.
  [Gukov-Vafa-Witten]

$$G_4 \in H^{(2,2)}_{\text{primitive}}(CY_4)$$

- $3D$ $N = 1$ supersymmetry:
  [Prins-Tsimpis]

$$G_4 = c\left(J \wedge J + \frac{3}{2}\text{Re}\Omega\right) + H^{(2,2)}_{\text{primitive}}$$

- If $CY_4 = K3 \times K3$, $N = 4, 2, 1$ possible.
  [Dasgupta-Rajesh-Sethi, Prins-Tsimpis]

- Metric matters! Flux gives mass to some metric moduli
  [GVW]

What happens to our $(2,0) \leftrightarrow$ heterotic string story?

- Take $\mathcal{M}_8$ to be conformally $K3 \times K3$
- Wrap M5 on one K3
- In background flux, H_3 gets shifted

$$\mathcal{H}_3 = H_3 - C_3, \quad dC_3 = G_4$$

- Analogous to $\mathcal{F} = F - B$ in DBI action
- Upon reduction on $K3$:

$$\mathcal{H}_3 = \sum_{i=1}^{19} \left(\partial_- X^i - A_-^i\right) \omega_i^- + \sum_{i=1}^{3} \left(\partial_+ X^i - A_+^i\right) \omega_i^+$$

- Theory is now interacting and can have reduced supersymmetry
- Should flow to heterotic dual of M-theory background
- Torsional, T^3-fibered non-Kähler spaces
  [McOrist-Morrison-Sethi]
- If M5 wrapped more general Σ_4, generalize MSW string
  [Maldacena-Strominger-Witten]

Lessons:

- The choice of flux changes the theory on the string
- The string landscape implies a landscape of $2D$ theories

How to incorporate flux more generally?

- Generally turns on operators charged under all of

$$SO(p,1) \times SO(D-p-1)_R$$

- Could enumerate and supersymmetrize—painful?
- Brane effective actions already include flux and supersymmetry

Brane physics $\to$ field theories

- In regime of validity, brane effective actions are Green-Schwarz-like
- Specify embedding into target superspace
- For M5-brane:

$$Z: \;\; \Sigma_{6|0} \to \mathcal{M}_{11|32}, \quad Z^A(\sigma) = \big(X^M(\sigma), \Theta^a(\sigma)\big)$$

- Worldvolume degrees of freedom are Z^A and two-form B_2
- Gauge symmetries make not all of Z^A physical

$$\phi^I \in \Gamma\left[\mathcal{N}\right], \quad \Theta \in \Gamma\left[\mathcal{S}^-\left(T\Sigma\right) \otimes \mathcal{S}\left(\mathcal{N}\right)\right]$$

Supersymmetries:

- Bulk superisometries generated by (fluxed-)Killing spinor:

$$\delta_\epsilon \phi^I = \bar{\epsilon}\Gamma^I \Theta, \quad \delta_\epsilon \Theta = \epsilon$$

- Local, kappa symmetry

$$\delta_\kappa \phi^I = -\left(\delta_\kappa \overline{\Theta}\right)\Gamma^I \Theta, \quad \delta_\kappa \Theta = (1+\Gamma_\kappa)\kappa,$$

- κ is arbitrary spinor field on Σ, $\Gamma_\kappa = \Gamma_\kappa(Z,\mathcal{H})$ satisfies

$$\text{tr}\Gamma_\kappa = 0, \quad \Gamma_\kappa^2 = 1$$

- Worldvolume has global supersymmetry ϵ only if [Becker-Becker-Strominger]

$$\delta_\kappa \Theta + \delta_\epsilon \Theta = 0 \quad \Longrightarrow \quad (1-\Gamma_\kappa)\epsilon = 0$$

M5-brane wrapping supersymmetric cycles w/o flux: [Gauntlett]

Calibration	World-Volume	Supersymmetry
SLAG	$\mathbb{R}^{1,3} \times (\Sigma_2 \subset CY_2)$	8, $\mathcal{N}$=2 d=4
	$\mathbb{R}^{1,2} \times (\Sigma_3 \subset CY_3)$	4, $\mathcal{N}$=2 d=3
	$\mathbb{R}^{1,1} \times (\Sigma_4 \subset CY_4)$	2, (1,1) d=2
	$\mathbb{R} \times (\Sigma_5 \subset CY_5)$	1
	$\mathbb{R}^{1,1} \times (\Sigma_2 \subset CY_2) \times (\Sigma_2' \subset CY_2')$	4, (2,2) d=2
	$\mathbb{R} \times (\Sigma_2 \subset CY_2) \times (\Sigma_3 \subset CY_3)$	2
Kähler	$\mathbb{R}^{1,3} \times (\Sigma_2 \subset CY_3)$	4, $\mathcal{N}$=1 d=4
	$\mathbb{R}^{1,1} \times (\Sigma_4 \subset CY_3)$	4, (4,0) d=2
	$\mathbb{R}^{1,1} \times (\Sigma_4 \subset CY_4)$	2, (2,0) d=2
$\mathbb{C}$-Lag	$\mathbb{R}^{1,1} \times (\Sigma_4 \subset HK_2)$	3, (2,1) d=2
Associative	$\mathbb{R}^{1,2} \times (\Sigma_3 \subset G_2)$	2, $\mathcal{N}$=1 d=3
Co-associative	$\mathbb{R}^{1,1} \times (\Sigma_4 \subset G_2)$	2, (2,0) d=2
Cayley	$\mathbb{R}^{1,1} \times (\Sigma_4 \subset Spin(7))$	1, (1,0) d=2

TABLE 4. The different ways in which fivebranes can wrap calibrated cycles and the amount of supersymmetry preserved.

What are the generalized calibrations?

Make two simple assumptions,

$$\mathcal{H}_3\Big|_{\Sigma} = 0, \quad G_4\Big|_{\Sigma} = 0.$$

Then, calibrated cycles are generalized calibrated cycles!

Calibration condition gives BPS bound [Becker-Becker-Strominger]

$$\int_{\Sigma} \bar{\epsilon}\,(1 - \Gamma_\kappa)^\dagger\,(1 - \Gamma_\kappa)\,\epsilon \geq 0.$$

Without flux, supersymmetric cycles are minimal submanifolds.

Only flux supported on Σ can modify this

What is the lesson?

- Generalized calibrations are as ubiquitous as flux backgrounds
- Flux backgrounds are ubiquitous and *qualitatively* change the physics (e.g. amount of SUSY)
- There should be a landscape of field theories to reflect this
- Combining calibration condition and 11-dimensional SUSY:

$$\nabla_\mu \epsilon + \frac{1}{12}\Big(\big\{\Gamma_\kappa, \Gamma_\mu \not{G}_4\big\} - 3\big\{\Gamma_\kappa, (\not{G}_4)_\mu\big\}\Big)\epsilon = 0$$

- G-structures for field theories

Back to the M5:

To leading order in a momentum expansion

$$\delta B_{\mu\nu} = -i\bar{\epsilon}\gamma_{\mu\nu}\Theta$$
$$\delta\phi^I = -i\bar{\epsilon}\Gamma^I\Theta,$$
$$\delta\Theta = -\frac{1}{2}D_\mu\phi^I\gamma^\mu\Gamma_I\epsilon - \frac{1}{24}G_{\mu KLM}\epsilon^{KLMI}{}_J\phi^J\gamma^\mu\Gamma_I\epsilon - \frac{1}{24}H^+_{\mu\nu\rho}\gamma^{\mu\nu\rho}\epsilon.$$

Back to the M5:

To leading order in a momentum expansion

$$\begin{aligned} \delta B_{\mu\nu} &= -i\bar{\epsilon}\gamma_{\mu\nu}\Theta \\ \delta\phi^I &= -i\bar{\epsilon}\Gamma^I\Theta, \\ \delta\Theta &= -\frac{1}{2}D_\mu\phi^I\gamma^\mu\Gamma_I\epsilon - \frac{1}{24}G_{\mu KLM}\epsilon^{KLMI}{}_J\phi^J\gamma^\mu\Gamma_I\epsilon - \frac{1}{24}H^+_{\mu\nu\rho}\gamma^{\mu\nu\rho}\epsilon. \end{aligned}$$

- D_μ is connection on normal bundle:

$$D_\mu\phi^I = \partial_\mu\phi^I - A^I_{\mu J}\phi^J$$

- Flux-modified connection

$$D^{(G)}_\mu\phi^I = D_\mu\phi^I + \frac{1}{12}G_{\mu KLM}\epsilon^{KLMI}{}_J\phi^J$$

Imply the equations of motion:

$$0 = H^{-}_{\mu\nu\rho} - \phi^I G^{-}_{I\mu\nu\rho},$$
$$0 = \gamma^\mu D^{(G)}_\mu \Theta - \frac{1}{24} G^{-}_{\mu\nu\rho I} \gamma^{\mu\nu\rho} \Gamma^I \Theta,$$
$$\gamma^\mu D^{(G)}_\mu \Theta := \left(\gamma^\mu \nabla_\mu - \frac{1}{4} A^{IJ}_\mu \Gamma_{IJ} + \frac{1}{48} G_{\mu IJK} \epsilon^{IJKLM} \Gamma_{LM} \right) \Theta.$$

- The ϕ^I equation is (for now) undetermined.
- Same algebra appeared in [Bergshoeff-Sezgin-van Proeyen, Cordova-Jafferis]
- Coupled $(2,0)$ tensor multiplet to off-shell conformal supergravity
- Auxiliary fields $\leftrightarrow$ fluxes [Triendl]

Questions:

- What is the relation to the off-shell approach?
- If they are the same, where is the landscape? If they aren't what is different?
- Can we relax the on-shell constraints?
- What can we learn about torsional heterotic solutions? Can we localize and calculate elliptic genera?
- Supersymmetric deformations of generalized calibrations $\rightarrow$ generalized geometry?
- Relation to higher-form symmetries?