Enumerating and exploring the sets of elliptic Calabi-Yau threefolds and fourfolds

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0. Intro
1. Classifying $\text{EFFY}_3$'s (start at large $h^{2,1}$)
2. Sampling $\text{EFFY}_4$'s
3. Physics:
   - How does F-theory $\rightarrow$ Standard model.

\section{Classifying compact CY $n$-folds ($K \sim 0$ up to twist)}

"data" on CY 3-folds

Kreuzer-Skarke $\sim 500$ million

$\rightarrow$ 30,000 Hodge number pairs
Physics of F-theory

Methodology for systematic analysis of EFCY's

a) Classify Bases

\[ E \rightarrow X \times CY_3, CY_4 \]

\[ B_{4,3} \]

b) "tune" Weierstrass model

\[ y^2 = x^3 + fx + g \]

\[ f \in \mathbb{P}(0(4k)) \]

\[ g \in \mathbb{P}(0(6k)) \]

\[ \rightarrow \text{different topological types of EFCY's on } B \]

Preview

- Systematic classification: "most" known CY's @ large h's are elliptic

- Toric bases B → representative sample of known allowed EFCY3

- Structure for 4-fold is highly parallel

- Modular structure from local "units"
§1. Kodaira/Classified Singularities in Fibrations

Tate

\[ y^2 = x^3 + fx + g \]
\[ 4f^3 + 27g^2 \]

Type: \( \text{ad}_D \cdot \text{ad}_g \cdot \text{ad}_\Delta \)

In: 0 0 0 n

\( \hat{A}_{n-1} \rightarrow \text{SU}(n) \)

\[ \begin{array}{ccc}
1 & 2 & 3 \\
4 & 5 & 10 \\
6 & 12
\end{array} \]

Feynman diagram:

Weinbass models \( \rightarrow \) Physical theories

\( \text{CY} 3 \rightarrow 6 \text{D} \)
\( \text{CY} 4 \rightarrow 4 \text{D} \)

Column 1 \( \rightarrow \) gauge group \( \mathbb{G}_N \)

Column 2 \( \rightarrow \) representation \( R \) of \( \mathbb{G}_N \)

"mixture"
Gross: finite # of $E(4)Y(3)'s$ up to birational automorphs.

Plan: study $B_2$ 

Minimal: $-1$ curve $\longrightarrow S$ 

Grassi: minimal bases which support $E(4)Y(3)'s$: \( P^2, \mathbb{F}_m, \mathbb{M}_m \) 

Enriques

Program: construct all $E(4)Y(3)'s$ by study with 

$P^2, \mathbb{F}_m, \mathbb{M}_m (m \leq 12)$

Blowup, tune moduli in Weierstrass model.

Useful tool: "Non-Higgsable clusters"

\[ C \cong P^1 \quad \text{N}(C) = 0 \]

\[ C \in B_2 \]

\[ \Rightarrow \] force a nontrivial singularity on $C$ if $n > 2$.

E.g., 

\[ (k+\epsilon) \cdot C = 2 - 2 = -2. \]

\[ -12 \]

\[ -12 \]

\[ -K \cdot C = -10 \]

\[ \Rightarrow C \text{ rigid, } C \text{ is a conpact of } -K. \]

\[ -4K = 4C + \text{eff} \quad \Rightarrow \text{ type } II \text{ on } C \]

\[ -6K = 5C + Y\text{eff} \quad \text{(Eg gauge group)} \]
\[ m = 3 \quad \cdots \quad m = 12 \]

\[ \text{strictly restricts } B_2 \]

Take \( B_2 \), general Calabi-Yau EF on \( B_2 \)

\[ h''(x) = h''(B) + r + 1, \quad r = \text{rank}(G) \]
\[ h''(x) = h''(B) + r + 1 \]

\[ h_{21} = 272 - 29(h''(B) - 1) + \dim C \cdot \dim R \]

\[ h'' = 2 + 8 + 1 \]

\[ h_{21} = 272 - 29 + 248 = 491 \]

For toric \( \sim 60,000 \), via blow-up

\( T^* \)-bases \( \sim 160,000 \)

toric base \( \to \) good sample
4-folds $\rightarrow$ look for toric 3-fold
alt. $\mathbb{P}^2$ $\rightarrow$ Start with $\mathbb{P}^3$

\[ \downarrow \]
$\mathcal{F}$, $\mathcal{F}_2$

\[ \rightarrow \]\hspace{1cm} Blow up at points or cones

\[ \downarrow \]
\hspace{1cm} Blow down at divisors

- Monte Carlo on graph
- prob($v_i$) $\propto$ Neighbors($v_i$)

Hodge Sheaf for 4-fold

(ES 3.05,143)
NHC for 3-f.w. bases:
loops, branching

only 5 products

\[
\text{SU}(2) \times \text{SU}(3) = \text{SU}(3) \times \text{SU}(3)
\]

Monte Carlo analysis

- \( N(\bar{h}''(B) = h) \) peaks at \( \sim h'' = 8.2 \pm 6 \)
- Total \# \( (= Z\bar{N}(h)) \sim 10^{48} \pm 2 \)
- # factors in \( C \sim \text{linear in } h''(B) \sim 0.35 h''(B) \)
- typical group \( \text{SU}(2)^19 \times C_2 \times \text{SU}(3)^2 \times \text{SU}(8) \times F_4^3 \)
\( \sim 10\% \) of connected pairs are \( SU(3) \times SU(2) \)

- divisors 6 singlets, 2 pairs, 2 by 2 by 2

... (biggest is typically 16)

... biggest cluster found: 37 factors

\[ \text{(complexified theory)} \]

\( \text{§3 Physics} \)

\( SM: \quad SU(3) \times SU(2) \times U(1) \)

How do we get from \( f \rightarrow \overline{f} \)?

1) "Tune" in \( B \) without using \( N \)s

- \( \text{e.g.} \; \mathbb{R}^3 \) can tune \( SU(5) \) "GUT"
- Cost in tuning
- Hard to tune \( SU(5) \) when a lot of \( N \)s

\[ \begin{align*}
SU(2) & \quad \text{IV} \\
SU(3) & \quad \text{III} \\
SU(5) & \quad \text{I} \\
R & \quad \text{IV}
\end{align*} \]

2) Get \( SU(3) \times SU(2) \) from \( \text{NHC} \)

(70\% have such a pair)

3) Break a \( \text{NHC} E_8, E_7, E_6 \) with fluxes
\[ M(\text{CY}4') \]

Superpotential

(\text{topological fluxes})

\text{Chain of } F \in H_4(X)

\text{Flux vacua:}

\# \text{flux vacua (fixed base)} \sim 10^3 \cdot (x)

\text{(253, 30342)}.

\text{B}_2 \cong \mathbb{R}_2

\text{CY}4

\text{CY}3

\text{N(vacua for this point)} \sim 10^{273000}

\text{Next we down in much smaller: } \sim O(10^{-3000})

Suggests: all but a fraction \(10^{-3000}\) of flux vacua come from \(M_{\text{max}}\).
\[ G = E_8^9 \times F_4^8 \times (G_2 \times SU(2)) / 6 \]

Would have to get \( SU(3) \times SU(2) \times U(1) \) by flux breaking \( E_8 \).

\( \sim \frac{1}{12} \) of flux to be "tuned in"

Prediction for dark matter: roughly 30 decoupled DM species: \( G_2 \times SU(1) \)

\( \rightarrow F_4 \)

\( \rightarrow E_8 \)

with some breaking by flux.