DRM 9/6/18
Supersymmetry & Seiberg Witten Theory

4D, $\mathcal{N}=2$ $SU(2)$ gauge theory

(no matter) following

Seiberg - Witten hep-th/9407087

<u>$\mathcal{N}=2$ multiplets</u>:

$\mathcal{N}=2$ vector: transform in adjoint rep of G
- gauge field A_μ
- two Weyl fermions λ, ψ
- scalar ϕ

$G = su(2)$

<u>$\mathcal{N}=2$ hypermultiplet</u>:

transform in a rep of G.
- two Weyl fermions ψ_q, $\psi^\dagger_{\tilde q}$
- two scalars $q, \tilde{q}^\dagger$

In $\mathcal{N}=1$ terms

$\mathcal{N}=2$ vector $=$ ($\mathcal{N}=1$) vector $+$ ($\mathcal{N}=1$) chiral

$\mathcal{N}=2$ hyper $= 2\,\mathcal{N}=1$ chirals

<u>Classical R symmetries</u>:

different actions on fermions
- $SU(2)_R$ anomaly-free acts on $\{q, q^\dagger\}$
- $U(1)_R$ acts on ϕ

This $U(1)_R$ is (almost always) anomalous.

$G = SU(N_c)$ and N_f "fundamental" hypermultiplets

$\Rightarrow$ anomaly free subgroup is $\mathbb{Z}_{4N_c - 2N_f} \subseteq U(1)_R$

<u>Classical potential for $\mathcal{N}=2$ theory</u>:

(g coupling constant)

$$V(\phi) = \frac{1}{g^2} \operatorname{Tr}[\phi, \phi^\dagger]^2$$

$V(\phi) = 0$ can happen if

$$[\phi, \phi^\dagger] = 0$$

$\longrightarrow$ space of solutions (vacuum degeneracy)

(ϕ takes values in the algebra)

$G = SU(2)$, ϕ is conjugate to

$$\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$u = \frac{1}{2} a^2 = \operatorname{Tr} \phi^2$

is the natural parameter.

$a \neq 0 \Rightarrow su(2)$ broken to commutant of $\frac{1}{2} a \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ which is $U(1)$

(Weyl $su(2)$ $a \mapsto -a$)

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provide the parameter to a field

A: ($\mathcal{N}=2$) vect multi whose scalar comp is a

Terms in SUSY low-energy theory involve a holo function

$$\mathcal{F}(A).$$

In $\mathcal{N}=1$ language of the lagrangian:

$$\frac{1}{4\pi} \mathrm{Im} \left[\int d^4\theta \, \frac{\partial \mathcal{F}(A)}{\partial A} \bar{A} + \int d^2\theta \, \frac{1}{2} \frac{\partial^2 \mathcal{F}(A)}{\partial A^2} W_\alpha W^\alpha \right]$$

Structure of the theory is encoded in $\mathcal{F}(A)$

$$\tau_{cl} = \frac{\theta}{2\pi} + i \frac{4\pi}{g^2}$$

Kahler potential for metric in space of scalars:

$$K = \mathrm{Im}\left(\frac{\partial \mathcal{F}}{\partial A} \bar{A} \right)$$

$$ds^2 = \mathrm{Im} \frac{\partial^2 \mathcal{F}(a)}{\partial a^2} \, da \, d\bar{a}$$

= kahler metric on the space of vacua,

Features of the quantum theory:

1) If $|a| \gg 0$, theory is "asymptotically free" (weakly coupled)

2) Vacuum degeneracy cannot be removed quantum mechanically since $\mathcal{N}=2$ supersymmetry does not permit an $\mathcal{N}=1$ superpotential

3) classical value of $\mathcal{F}$ is

$$\mathcal{F}(A) = \frac{1}{2} \tau_{cl} A^2$$

$$\tau(a) = \frac{\partial^2 \mathcal{F}}{\partial a^2}$$

$$\mathcal{F}_{\text{one-loop}} = i \frac{1}{2\pi} A^2 \ln \frac{A^2}{\Lambda^2}$$

(Λ = dimensional scale)

($\mathcal{N}=2$ SUSY $\Rightarrow$ 1 loop exact)

$$\mathcal{F} = i \frac{1}{2\pi} A^2 \ln \frac{A^2}{\Lambda} + \underbrace{\sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{A} \right)^k A^2}_{\text{instanton corrections}}$$

$$\mathcal{F}_1 \neq 0$$

__Duality__: $ds^2 = \mathrm{Im}\,\tau(a)\, da\, d\bar{a}$

for large $|a|$

- $\tau(a) \approx i\left(\dfrac{\ln\left(\frac{a^2}{\Lambda^2}\right)+3}{\pi}\right)$
- $\tau(a)$ multivalued
- ds^2 is not pos. definite $\forall a$.

Let $a_D = \dfrac{\partial F}{\partial a}$

$$ds^2 = \mathrm{Im}(da_D\, d\bar{a})$$
$$= -\frac{i}{2}\left(da_D\, d\bar{a} - da\, d\bar{a}_D\right)$$

__Observation__:

$a \longleftrightarrow -a_D$ gives another description of the same theory.

Note: $a_D \longrightarrow a_D + ba$, $a \longrightarrow a$ leaves the invariant

$$\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a_D \\ a \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} a_D \\ a \end{pmatrix}$$

$$\begin{matrix} SL(2,\mathbb{R}) \\ \cup \\ SL(2,\mathbb{Z}) \end{matrix}$$
broken to by BPS particles

$Z = a n_e + a_D n_m$

Light magnetic $a_D = 0$

(require at least 4 singularities to give multivaluedness)

Light electric $a = 0$

$a \to -a$ action of Weyl (SU(2))

$u = \frac{1}{2} a^2$