provide the parameter $k$ a field $A \propto (N=2)$ with non-abelian scalar coupling $\alpha$.

Terms in SUSY low-energy theory involve a holomorphic function $F(A)$.

In $W=1$ language

The Lagrangian:

$$L = \text{Im} \left[ \int d^2 \theta \frac{\partial^2 F(A)}{\partial A^2} A \right] + \int d^2 \theta \frac{\partial^2 F(A)}{\partial A^2} \bar{W} \bar{W}$$

Structure of the theory is encoded in $F(A)$

$$L_{cl} = \frac{\alpha}{2 \pi} \frac{4 \pi}{g^2}$$

Kaluza potential for metrics in space of scalars:

$$K = \text{Im} \left( \frac{\partial F(A)}{\partial A} \right)$$

$$k^2 = \text{Im} \frac{\partial^2 F(A)}{\partial A^2} \bar{d} \bar{d} a$$

= Kaluza metric in the space of vacua.

Features of the quantum theory:

1) if $|\alpha| \gg 1$, they are "asymptotically free"
   (weakly coupled)

2) vacuum decay cannot be removed quantum mechanically
   since $N=2$ supersymmetry does not permit a $W=1$ superpotential

3) classical value of $F$ is

$$F(A) = \frac{1}{2} \bar{z}_0 A^2$$

$$\bar{z}_0 = \frac{\alpha}{2 \pi} g^2$$

$$F_{\text{two-loop}} = i \frac{1}{16 \pi^2} A^2 \bar{K} \frac{\partial A^2}{A^2}$$

($\lambda =$ dimension scale)

($N=2$ string $\Rightarrow$ 1-loop exact)

$$F = i \frac{1}{16 \pi^2} \frac{\partial A^2}{A^2} \bar{K} \frac{\partial A^2}{A^2}$$

$$+ \int_{k_0}^\infty \frac{F(A)}{A^2} A^2$$

($k_0$ cutoff, conventions)

$$F = 0$$
Duality: \( ds^2 = \text{Int}(h) \text{d}a_\text{d}c \)

for large \( \ell \),

\( t(a) \propto \frac{h(\frac{\ell^2}{a}) + 3}{a} \)

- \( t(a) \) multiply\( k \)
- \( \text{Int}(h) \) is not pos. definite \( \forall a \),

\( \text{Int} \equiv \int_0^\infty \text{Int}(h) \text{d}a \)

\( ds^2 = \text{Int}(h) \text{d}a_\text{d}c \)

\( = -\frac{i}{2} \left( h_a \text{d}a_\text{d}c - \text{d}c \text{d}a \right) \)

Observe:

\( a \leftrightarrow -a \) gives another description of the same theory.

Note:

\( a_0 \rightarrow a_0 + b \) leaves the

\( a \rightarrow a \) invariant

\( \left( \begin{array}{c} 0 \ 1 \end{array} \right) \left( \begin{array}{c} a_0 \\ a \end{array} \right) = \left( \begin{array}{c} 0 \ 1 \end{array} \right) \left( \begin{array}{c} a_0 \\ a \end{array} \right) \)

\( SL(2, \mathbb{R}) \) broken to

\( SL(2, \mathbb{R}) \) by BPS particles.