Geometry, Topology, and Physics Seminar

Higher Chow groups and the Abel-Jacobi map

Dave Morrison
UCSB

Friday, December 8, 2017, 4:00 p.m.
Room 6635 South Hall

Abstract: The original Abel-Jacobi map from a complex algebraic curve to its Jacobian was generalized by Griffiths to the following construction, also called the Abel-Jacobi map. The $p^{th}$ intermediate Jacobian of an algebraic variety $X$ is the complex torus

$$(H^{2p+1,0}(X) + H^{2p,1}(X) + \ldots + H^{p,p}(X))^*/H_{2p+1}(X, \mathbb{Z}),$$

where the integer $(2p + 1)$-cycles define linear maps on de Rham cohomology via integration.

Consider an algebraic cycle $Z$ on $X$ of (complex) dimension $p$ which is homologous to zero. Then there is a $(2p + 1)$-chain $\Gamma$ whose boundary is $Z$. We can integrate $(2p + 1)$-forms over $\Gamma$ and get answers which are well-defined up to integrals over $(2p + 1)$-cycles. Thus, $Z$ determines a well-defined point in the $p$th intermediate Jacobian. The image only depends on the rational equivalence class of $Z$, i.e., on the corresponding element of the Chow group. This is Griffiths’ version of the Abel-Jacobi map.

In this lecture, following a paper of Kerr, Lewis, and Müller-Stach, we will present a refinement of the Abel-Jacobi map which is relevant when $X$ has a natural mixed Hodge structure rather than a pure Hodge structure. It involves some higher Chow groups which were originally defined by Bloch.

This is an ingredient in my work in progress (with Jockers and Walcher) on higher Chow groups, mirror symmetry, and hyperbolic 3-manifolds.

This seminar is part of the NSF/UCSB ‘Research Training Group’ in Topology and Geometry. Information about future meetings can be found at http://www.math.ucsb.edu/~drm/GTPseminar/