Western Hemisphere Colloquium on Geometry & Physics 03/08/21 1.

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2-Group Global Symmetry in Quantum Field Theory

main ref's: 2009.00138, 1802.04790 w/ C. Cordova & K. Intriligator

Familiar ex. of flavor symmetry in QFT: $U(1)$ flavor symmetry (e.g. baryon number in QCD).

$\longrightarrow$ Noether current j_μ, $\partial^\mu j_\mu = 0$

Convention: this is an ordinary / 0-form global symmetry $U(1) = U(1)^{(0)}$

$j^{(1)} = j_\mu dx^\mu$ is its 1-form current.

Operator conservation eq.: $d * j^{(1)} = 0.$

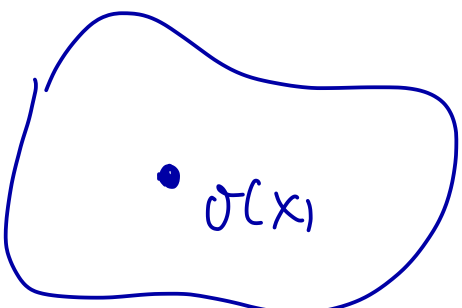
Can define topological charge operators. Consider a 4d QFT on $\mathcal{M}_4$ (e.g. $\mathcal{M}_4 = \mathbb{R}^4$).

$$Q(\Sigma_3) = \int_{\Sigma_3} * j^{(1)}$$

conservation of $j^{(1)}$ $\Rightarrow$ topological invariance of $Q(\Sigma_3)$

Σ_3: codim-1 in $\mathcal{M}_4$

Charged objects = local operators $\mathcal{O}(x)$



$Q(\Sigma_3)$ $\quad\Sigma_3$

$$= q_0 \, \mathcal{O}(x)$$

q_0: $U(1)^{(0)}$ charge of $\mathcal{O}(x)$

$\mathcal{O}(x)$ can create point particles w/ $U(1)^{(0)}$ charge.

One more layer needed: (standard for global symmetries in QFT) Couple $j^{(1)}$ to a non-dynamical/classical source:

$$\Delta S = \int j^{\mu} A_{\mu} = \int A^{(1)} \wedge * j^{(1)}$$

Standard fact:

$$\left\{\begin{array}{c}\text{current}\\ \text{conservation:}\\ d * j^{(1)} = 0\end{array}\right\} \Longleftrightarrow \left\{\begin{array}{c}\text{partition function}\\ Z[A^{(1)}] \text{ invariant}\\ \text{under } A^{(1)} \to A^{(1)} + d\lambda^{(0)}\end{array}\right\}$$

$U(1)^{(0)}$ background gauge transformations, i.e. $A^{(1)}$ is an ordinary $U(1)^{(0)}$ background gauge field.

Recently this familiar story has been generalized in important ways, e.g.

higher form ("generalized") global symmetries [Kapustin, Seiberg; Gaiotto, Kapustin, Seiberg, Willett]

Simplest 4d example:

$U(1)^{(1)}$ = continuous 1-form global symm.

$J^{(2)}$ = conserved 2-form current

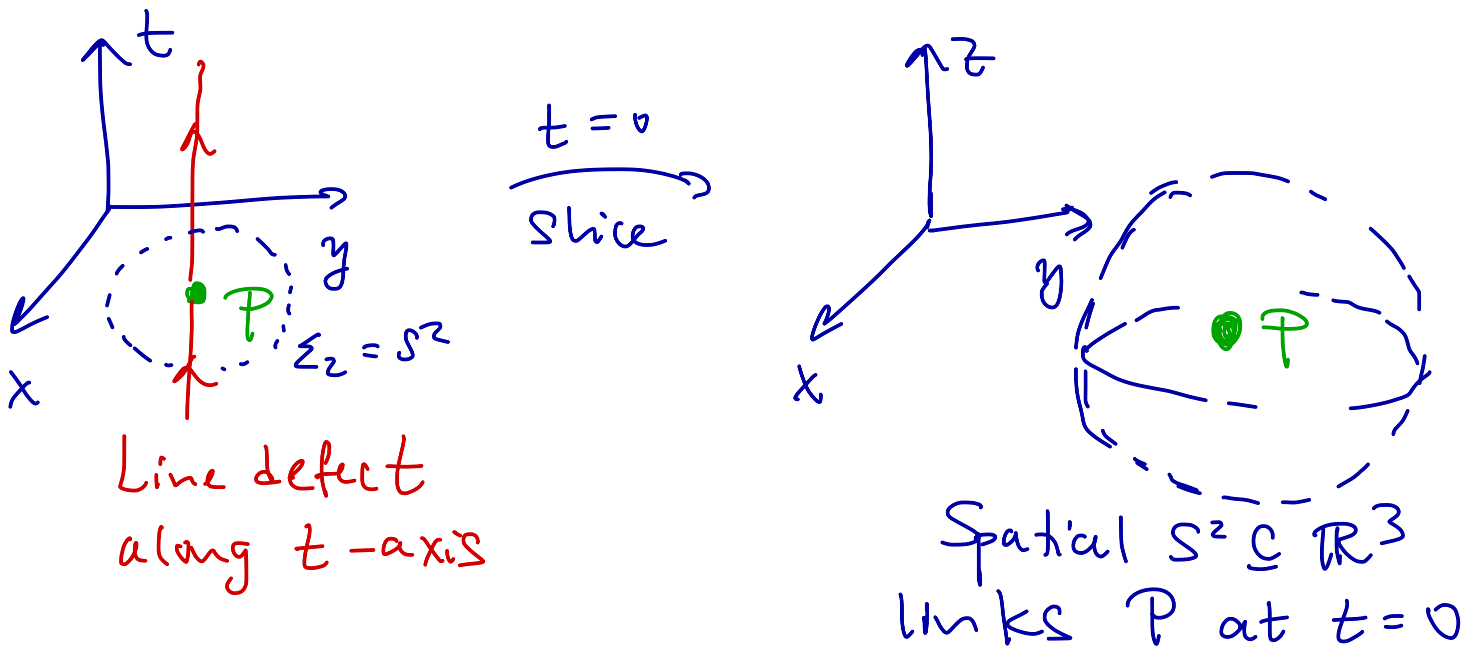
$$d * J^{(2)} = 0 \quad (\partial^\mu J_{[\mu\nu]} = 0)$$

Topological surface operators (codim-2)

$$Q(\Sigma_2) = \int_{\Sigma_2} * J^{(2)}$$

$\Sigma_2 \subset M_4$

Charged objects ~ lines linked by Σ_2.



The charged line defects/operators can create dynamical strings charged under $U(1)^{(1)}$.

Simplest 4d example: free Maxwell theory

$$f^{(2)} = d a^{(1)}, \qquad d f^{(2)} = 0 \quad \text{(Bianchi id.)}$$

dynamical U(1) connection $\qquad d * f^{(2)} = 0 \quad$ (e.o.m.)

$\Rightarrow$ 2 1-form symmetries

$$U(1)^{(1)}_E : J_E^{(2)} \sim f^{(2)}, \quad Q_E(\Sigma_2) = \int_{\Sigma_2} * J_E^{(2)} \sim \int_{\Sigma_2} * f^{(2)}$$

electric charge

$$U(1)^{(1)}_M : J_M^{(2)} \sim * f^{(2)}, \quad Q_M(\Sigma_2) = \int_{\Sigma_2} * J_M^{(2)} \sim \int_{\Sigma_2} f^{(2)}$$

magnetic charge

Charged op's: Wilson & 't Hooft lines
(electric) (magnetic)

Final Step: Couple $J_E^{(2)}$, $J_M^{(2)}$ to background gauge fields ~ 2-form B-fields.

$$\Delta S = \underbrace{\int B_E^{(2)} \wedge * J_E^{(2)}}_{\sim \int B_E^{(2)} \wedge * f^{(2)}} + \underbrace{\int B_M^{(2)} \wedge * J_M^{(2)}}_{\sim \int B_M^{(2)} \wedge f^{(2)}}$$

Standard "BF" or Green-Schwarz (GS) coupling.

Again: $d * J_{E,M}^{(2)} = 0$

$\Longleftrightarrow$ invariance under 1-form background gauge transformations:

$$B_{E,M}^{(2)} \longrightarrow B_{E,M}^{(2)} + d\Lambda_{E,M}^{(1)}$$

1-form gauge parameters.

This is the beginning of a rich story, with many recent results.

Key point: Symmetries are key in analyzing QFTs (esp. at strong coupling). So more, and new kinds of, symmetries are better.

<u>This talk</u>: yet a further generalization of global symmetry to higher n-group symmetries; simplest case = 2-group.

<u>Basic intuition</u>: in many examples the ordinary 0-form symmetries and the higher-form symmetries do not talk to each other. Higher group symmetries arise when they do.

<u>A prototypical example</u>: 4d QED_{N_f}, i.e. $U(1)$ Maxwell theory coupled to N_f Dirac electrons of mass $m=0$ and charge $q=1$, $\Psi_D^i = \begin{pmatrix} \psi_\alpha^i \\ \bar{\chi}^{\dot{\alpha}}_i \end{pmatrix}$ $i=1,\dots,N_f$

classical symmetries:

	$U(1)_{gauge}$	$U(1)_A^{(0)}$	$SU(N_f)_L^{(0)}$	$SU(N_f)_R^{(0)}$
ψ_α^i	1	1	$\square$	$\emptyset$
$\chi_{\alpha i}$	-1	1	$\emptyset$	$\square$

<u>note</u>: $\chi_{\alpha i}$ is a $\bar{\square}$ of $SU(N_f)^{(0)}_{diagonal}$.

QED retains the $U(1)^{(1)}_M$ magnetic 1-form symmetry since $df^{(2)} = 0$ still, but $U(1)^{(1)}_E$ is broken completely since $d * f^{(2)} \sim * j^{(1)}_E \neq 0$.

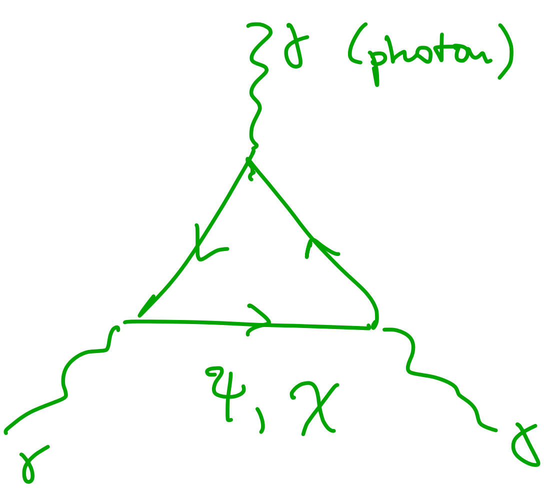
To discover the fate of the remaining symmetries, let us consider their perturbative anomalies ~ anomaly polynomial $I^{(6)}$

$$I^{(6)} = I^{(6)}_{\text{gauge}} + I^{(6)}_{\text{ABJ}} + I^{(6)}_{\text{global}} + I^{(6)}_{\text{mixed}}.$$

calculate via fermion Δ's

1.) $I^{(6)}_{\text{gauge}} \sim K_{\text{gauge}} \left(c_1(f^{(2)}) \right)^3$

$$K_{\text{gauge}} = q_\psi^3 - q_\chi^3 = 1 - 1 = 0.$$



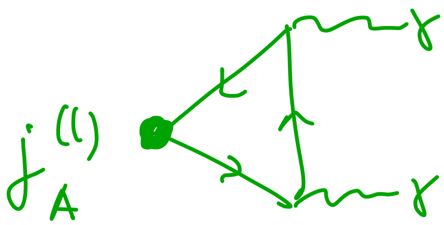
These pure gauge anomalies cancel ⟹ required for consistency of gauge thy.

$$\Longrightarrow \quad I^{(6)}_{\text{gauge}} = 0.$$

2.) $I^{(6)}_{ABJ} \sim K_{ABJ}\, c_1(F_A^{(2)})\, c_1(f^{(2)})^2$

$U(1)_A^{(1)}$ b.g. field.

maxwell field



This is the famous Adler-Bell-Jackiw (ABJ) axial anomaly

$$K_{ABJ} = \underbrace{1(1)^2}_{4} + \underbrace{1(-1)^2}_{\chi} = +2 \neq 0.$$

upshot: $d * j_A^{(1)} \sim \underbrace{K_{ABJ}\, f^{(2)} \wedge f^{(2)}}_{\text{non-zero operator (even when all b.g. fields are set to zero)}} \neq 0$

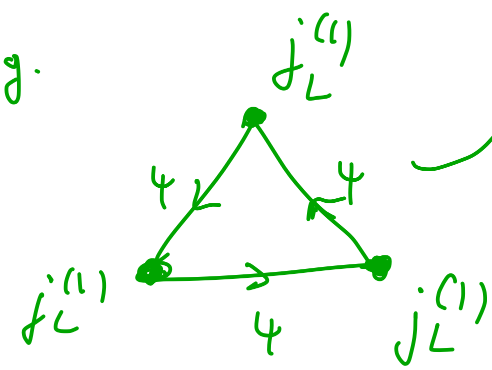
Thus $j_A^{(1)}$ is not conserved and we no longer consider the $U(1)_A^{(0)}$ symmetry.

3.) $I^{(6)}_{global} \sim \underline{c_3(F^{(2)}_{SU(N_f)_L})} - c_3(F^{(2)}_{SU(N_f)_R})$

only background fields

$\sim$ 't Hooft anomalies for global $SU(N_f)_L^{(0)}$ and $SU(N_f)_R^{(0)}$ symmetries.

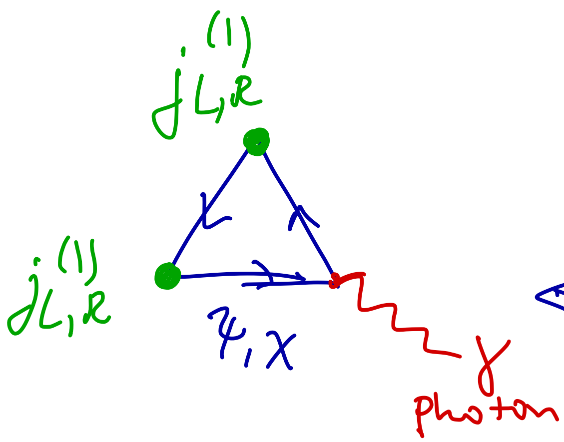
e.g.

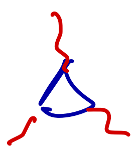
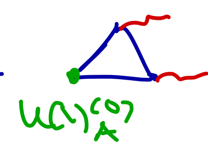


Key fact: 't Hooft anomalies do not break global symmetries; currents conserved as operators: $d*j_L^{(1)} = d*j_R^{(1)} = 0$ at separated points. But: subtle c-number violations at coincident points/of background gauge invariance.

4.) $$I^{(6)}_{\text{mixed}} \sim \left(c_2\left(F^{(2)}_{SU(N_f)_L}\right) - c_2\left(F^{(2)}_{SU(N_f)_R}\right)\right) \wedge \underbrace{c_1\left(f^{(2)}\right)}_{\text{dynamical } U(1) \text{ Maxwell field.}}$$

$SU(N_f)_L^{(1)} \times SU(N_f)_R^{(1)}$ b.g. fields



This diagram neither destroys gauge invariance (like ) nor is it an ABJ anomaly that destroys conservation of $j_L^{(1)}, j_R^{(1)}$ (like )

But the monodromy diagram still has interesting effect on the global symmetries:

$$d*j_{L,R}^{(1)} \sim \pm \underbrace{dA_{L,R}^{(1)}}_{\text{c-number}} \wedge \underbrace{c_1\left(f^{(2)}\right)}_{\text{operator} \sim *J_M^{(2)}}$$

in the absence of b.g. fields ($A^{(1)}_{L,R}=0$)
the $SU(N_f)^{(0)}_L \times SU(N_f)^{(0)}_R$ currents are
conserved operators: $d * j^{(1)}_{L,R} = 0$.

But their current algebra is deformed:

$$d * j^{(1)}_{L,R}(x)\, j^{(1)}_{L,R}(y) \sim \pm\, \partial\delta^{(4)}(x-y)\, f^{(2)}(y)$$

integrating this gives a (singular) OPE:

$$j^{(1)}_L(x)\, j^{(1)}_L(y) \sim \frac{J^{(2)}_M(y)}{(x-y)^4}\,, \quad j^{(1)}_R(x)\, j^{(1)}_R(y) \sim \frac{-J^{(2)}_M(y)}{(x-y)^4}$$

This fusion of two 1-form currents into a 2-form current is the defining feature of (continuous) 2-group global symmetry.

Background fields:

$$S \supseteq \int A^{(1)}_{L,R} \wedge * j^{(1)}_{L,R} + \int B^{(2)}_M \wedge \overbrace{* J^{(2)}_M}^{\sim f^{(2)}}$$

normally:

$$A^{(1)}_{L,R} \longrightarrow A^{(1)}_{L,R} + d_A \lambda^{(0)}_{L,R}$$

$$B^{(2)}_M \longrightarrow B^{(2)}_M + d\Lambda^{(1)}_M.$$

with 2-group symmetry, extra term:

$$B_M \longrightarrow B_M^{(2)} + d\Lambda_M^{(1)} + \frac{1}{4\pi}\, tr(\lambda_L^{(0)} dA_L^{(1)}) - \frac{1}{4\pi}\, tr(\lambda_R^{(0)}\, dA_R^{(1)})$$

needed to cancel RHS of $d * j_{L,R}^{(1)} \sim \pm\, dA_{L,R}^{(1)} \wedge f^{(2)}$.

Thus 2-group global symmetry $\Longleftrightarrow$ Green - Schwarz mechanism for background fields [Kapustin, ...]

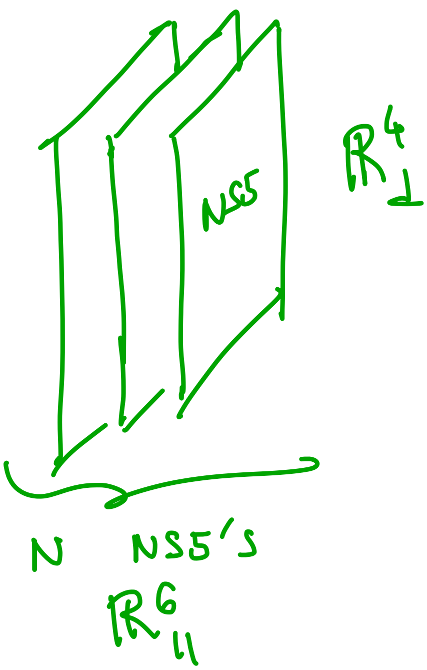
We have seen that anomalies of type $I^{(6)}_{mixed}$ inevitably give rise to 2-group global symmetry in 4d.

Remainder of talk: sketch some recent applications of 2-group symmetry to 6d Supersymmetric Little String Theories and SCFTs.

The $\mathcal{N}=(1,1)$ LST:

[Seiberg]

decoupling limit of NS5 stack in IIB



$(1,1)$ SUSY, $SU(2)_L \times SU(2)_R$ R-symmetry (from $\mathbb{R}^4_\perp$)

Q_L Q_R

$\xrightarrow{\text{IR}}$ 6d MSYM with gauge group $\mathfrak{g} = U(N)$. We will decouple the C.O.M. and take $\mathfrak{g} = SU(N)$.

Has $U(1)^{(1)}_I$ instanton 1-form symmetry:

$$J^{(2)}_I \sim * \, c_2(f^{(2)} \wedge f^{(2)}) \,, \quad d * J^{(2)}_I = 0$$

(present in <u>any</u> 6d gauge theory)

instantons $\sim$ little strings $\sim$ fundamental IIB strings

Compute (via $\square$ diagram):

$$I^{(8)}_{\text{mixed}} = N \left(c_2(L) - c_2(R) \right) \cdot c_2(f_{\mathfrak{g}}^{(2)})$$

$SU(2)_{L,R}$ background fields

dynamical $SU(N)$ gauge field

$\Longrightarrow$ 2-group symmetry!

This is generic in 6d gauge theories (with or without SUSY). We saw it happens in $\mathcal{N}=(1,1)$ LSTs with $SU(N)$ gauge group. We also checked other LST examples, and even more were checked by [del Zotto - Ohmori]

Emerging Picture: • all LSTs seem to have a $U(1)^{(1)}$ symmetry (little string charge)

• generically it is part of a 2-gp.

• [dZ-O] matched this structure for many T-dual pairs of LSTs (meaningful!)

What happens in 6d SCFTs?

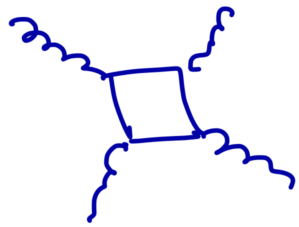
All SCFTs with gauge fields on their tensor branch have a GS coupling:

$$S_{GS} = i\sqrt{c}\int b^{(2)} \wedge \underbrace{c_2(f_g^{(2)})}_{*J_I^{(2)}}$$

dynamical S.D. 2-form in a (1,0) tensor-multiplet $(\phi, \psi_\alpha^i, b^{(2)})$

The GS coupling has many important roles: (14.)

(i) contributes $I^{(8)}_{GS} = +c\, c_2(f_g^{(2)})^2$ to $I^{(8)}$, which cancels $I^{(8)}_{1\text{-loop}} = -c\, c_2(f_g^{(2)})^2$ from matter



(ii) gauges $J^{(2)}_I \Rightarrow$ no longer global symmetry!

SCFT unitary rep. theory says that a conserved 2-form current is not allowed [CDI, 1612.00809].

$\Rightarrow$ {no $U(1)^{(1)}_I$ or 2-gp symmetries in unitary SCFTs} !

(iii) Superpartner of dilaton coupling

$\sim \sqrt{c}\, \phi\, \mathrm{tr}(f_g^{(2)} \wedge * f_g^{(2)})$, which makes YM term conformal ($\phi \sim$ dilaton)

The <u>absence</u> of 2-group symmetries in SCFTs has non-trivial consequences:

$$S_{GS} = i \int b^{(2)} \wedge \left(\sqrt{c}\, c_2(f_g^{(2)}) + \underbrace{\kappa\, c_2(R) + \cdots}_{\text{b.g. fields for global symmetries.}} \right)$$

dynamical

$$\Rightarrow \quad I^{(8)}_{GS} = c\,\left(C_2(f_g^{(2)})\right)^2 + k^2\,\left(C_2(R)\right)^2$$

't Hooft anomaly

$$+\, 2\sqrt{c}\; k\; \underbrace{C_2(f_g^{(2)})}_{\text{gauge}}\; \underbrace{C_2(R)}_{\text{global}}$$

<u>But:</u> in $I^{(8)}_{total} = I^{(8)}_{1\text{-loop}} + I^{(8)}_{GS}$ all mixed gauge-global terms must cancel: otherwise there would be a 2-gp in the SCFT!

<u>Thus:</u> Can calculate coefficients like k (and thus 't Hooft anomalies of the SCFT) by insisting that mixed terms in $I^{(8)}_{GS}$ cancel those in $I^{(8)}_{1\text{-loop}}$. This justifies the procedure of [Ohmori, Shimizu, Tachikawa, Yonekura] for computing 't Hooft anomalies of SCFTs from the low-energy spectrum on their tensor branch.

<u>Application:</u> the a-theorem in 6d

Weyl anomaly $\quad \langle T^{\mu}_{\mu} \rangle \sim a\,E_6 + \cdots$

Cardy Conjecture ("a-theorem")

$$\Delta a = a_{UV} - a_{IR} > 0$$

under RG: $CFT_{UV} \rightarrow CFT_{IR}$

recall: • intuition: $a \sim$ # effective d.o.f. along RG flow (16.)

• proven in 2d [Z] and 4d [KS]

• no general proof in $d \geq 6$ (without SUSY)

Some time ago [CDI 1506.03807] we made progress for SUSY RG flows, all of which look like $SCFT_{UV} \longrightarrow \begin{pmatrix}\text{Moduli}\\ \text{Space}\end{pmatrix}_{IR}$ (no susy-preserving relevant ops.)

we showed: $\Delta a \sim b^2 > 0$ (a-thm)

where $b \sim$ GS coefficients $\sim k\, b^{(2)} \wedge c_2(R)$ and $\sim b^{(2)} \wedge p_1(T)$ on tensor branch.

If IR theory is a free SCFT (only tensors + hypers, e.g. $\mathcal{N}=(2,0)$ SCFTs) then $a_{IR} > 0$

$$\Rightarrow \quad a_{SCFT} = \underbrace{a_{IR}}_{>0} + \underbrace{\Delta a}_{>0} > 0$$

It is expected that $a > 0$ in unitary CFTs (# d.o.f.), but it is distinct from $\Delta a > 0$, and no general pf. (e.g. in 4d $a > 0$ follows from [HM])

17.

In fact, there is a problem with vector multiplets:

$$a_{IR} = \frac{1}{210}\left(11 n_H + 199 n_T - 251 n_V\right)$$

formally (since vectors $\neq$ SCFT).

Enough vectors can lead to $a_{IR} < 0$ (happens in many examples! In fact generic.).

What about $a_{SCFT} > 0$? Only way out is that $\Delta a > |a_{IR}| > 0$ (*)
(stronger than a-thm).

Apply previous results: unitary SCFT $\longrightarrow$ $\longrightarrow$ no 2-gp symmetry $\longrightarrow$ no mixed gauge-global anomalies $\longrightarrow$ fix GS terms $\sim b^{(2)} \wedge C_2(R)$ and $b^{(2)} \wedge p_1(T)$ to compute

$$\Delta a \sim \frac{1}{|k_{(g^2)^2}|}\left(k_{g^2R^2} - k_{g^2p^2}\right)^2$$

with $I^{(8)}_{\text{1-loop}} \supset C_2(f_g)\left(k_{(g^2)^2} C_2(f_g) + k_{g^2R^2} C_2(R) + k_{g^2p^2} p_1(T)\right)$

The formula for Δa only depends on the massless <u>spectrum</u> on the tensor branch (no details of massive states needed)!

Bound Δa from below:

$$k_{g^2 R^2} = -h^\vee_g \,, \quad k_{g^2 p_2} = -2h^\vee_g + \cancel{(\text{hypers})}^{\,0}$$

$$k_{(g^2)^2} = -u_g + \cancel{(\text{hypers})}^{\,0} \qquad \mathrm{Tr}_{adj}(f_g)^4 \supseteq u_g \left(\mathrm{Tr} f_g^2\right)^2$$

Get a bound: $\Delta a > \dfrac{23 (h^\vee_g)^2}{u_g}$ ("universal": only $\sim g$)

Δa always wins! (check $\forall g$)

$$a_{IR,\,\text{vector}} \simeq -1.2\, \underbrace{\dim(g)}_{n_V}$$

<u>Thus</u>:

- prove $a_{SCFT} > 0$ for unitary 6d SCFTS!
- proof works for all ranks (here: rank 1)
- does <u>not</u> rely on detailed classification (general QFT result)
- Crucial: ability to compute GS couplings by enforcing <u>no 2-gp symm.</u>