Gauge theory in 3D and its boundary theories

3D Mirror symmetry has 2D mirror symmetry on boundary

"Coherent sheaf side" ? "Holomorphic Fukaya-Thy" 

"Rozansky-Witten theory"

piece gauge theory

topologically

What are gauged TFT's ?

On body of 2D toy gauge theory

Meta-theorem:

Prime 3D toy gauge theory \(\leftrightarrow\) RW theory for the

\(H^*\) Toda integrable system

\((L^*\text{ rem}
\)

\(K^*\) finite difft Toda system

Old challenge: extend down to points

KRS category in holomorphic symplectic manifolds?

\(D^Z\) of this = BTC DColh (X)

With structure constant

from hol st + sympl form.

I have a proposal for Integrable systems

(Based on Rel. field theory)
2. Toda integrable systems (Toda, Kostant...)

Bernt Markvickor, Finkel, Brinkovic

\[ G \text{ cyclic we group } \rightarrow H^G_\ast(\Sigma G) \leftarrow M^G_\ast \]

\[ G \text{ cyclic we group } \rightarrow K^G_\ast(\Sigma G) \leftarrow K^G_\ast \]

Hopf algebras Toda bases

Abelian

\[ \text{Group schemes over the base, symplectic manifolds,} \]

For \( G = T \), get \( C^T(T^* T^\mathbb{C}) \rightarrow C^T(T^* T^\mathbb{C}) \]

geom.

[Description in terms of Langland duality]

\[ T^V \]

\[ C^x(\Sigma G) \]

\[ \text{shearing of category } \]

\[ \text{Lagrangian support in Toda space.} \]

Categorie \( \nu \). Topological \( G \) action

With real Math properties, eg

How shevere collapses

CY condition?
3. Primeval 3D mirror symmetry – EM cleavages

Finite group monoids:

\[(\text{Vect} < F >, \ast) \equiv (\text{Rep}(F), \otimes)\]

Diagram:

\[
\begin{array}{ccc}
\mathbb{Z} & \xrightarrow{\text{Vect}(F)} & \text{Vect} \\
\text{BTC} & \xrightarrow{\text{Rep}(F)} & \text{Carr}
\end{array}
\]

\[\text{Vect} < F > \otimes \text{Rep}(F) = \text{Vect} \]

Point came full info so gave count equivalence.

Topological reps of G:

\[(\text{Vect} < G/\bar{G} >, \ast) \equiv (\text{Rep}(G/\bar{G}), \otimes)\]

\[(\text{Vect}(B^t), \ast) \equiv (\text{Rep}(B^t), \otimes)\]

\[(\mathbb{Z} - \text{module}, \otimes) \equiv (\mathbb{Z} \oplus \mathbb{Z}_{-1} - \text{module}, \otimes)\]

\[
\text{E}_2 \otimes \mathbb{H}^* \\
\text{Symp} \otimes \text{Sym}^{\ast-2}, \otimes
\]

\[
\text{braided stuff from 4mod from...}
\]
4. What is $\text{Rep}(G/G)$?

One answer is the CCC

$$G \times \text{Syng}^*, \ W = \mathfrak{h}(G) \otimes \mathfrak{g}^* \text{ as basis of } G$$

$$\text{ev}: \ g \rightarrow G \text{ exponential}$$

This is a tensor category

$$G = \iota: \ H \otimes W: \ i \otimes \langle \chi \mid \bar{z} \rangle$$

A might be a structure: pointwise in each that add that label

Morally: $DZ \xrightarrow{\delta} T^*T / \text{rational translations}$

Characters

$$= T^*TV.$$

Moral statement: Toda space = uncompleted $DZ$ of the tensor cat given by CCC.

If $A$ is an algebra a category with $G/G$ acting

$$L \xrightarrow{\phi} HZ^1(A), \ DGA morphism$$

$\text{Sections: } \text{defined from actin } \phi$

$$G \setminus \text{Riem}^*(W) \cong G/G \text{ as algebra or category}$$

need $\text{Rec}(\text{Maps allowed})$

not in linear

Eg $G/G$ acts on Vect by the quadratic form as $g \rightarrow T_g \phi$.
Other sanity checks,

\[ G = S^1 \text{ let it act trivially on } A \]

interesting braid path --- covering of the category

This defines a \( \mathbb{Z} \)-action on this category

\[(
\psi_{g,w}) \quad (\text{auto of } \text{Id}_M, e^w,)
\]

Thus \( \text{Tate } \mathbb{Z}_p \text{ category } = MF(A, W). \)

Very special case of FP completion using CCC.

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Fusion calculations: fibre of \( D^2 \) on the base CCC from cat

\[ \text{From } \Sigma \in g, \text{ centralizer } n \in G \]

\[ \text{Vect } \otimes \text{ Vect } = \text{ Vect } \otimes (\text{Rep}(\Sigma)) \otimes \text{ Vect } \]

\[ \text{Rep}(\text{vect}) \]

\[ = \text{ Vect } \otimes \text{ Rep}(H) = \text{ Vect}(\text{vect}^H) \]

\[ \text{Rep}(H) \]

Module cat

becomes \( \geq \) by monodromy rep as \( n \)

(for a closed gp \( N \))

\[ (\text{if } N = T) \]
5. What changes when adding matter?

Quantum rep. of $G$, $E$. $E_j = \mathbb{V} \oplus \mathbb{V}^\vee$

$G \times QH(E) \subset$ holomorphic Fukaya 2-cat 

When $E = \mathbb{V} \oplus \mathbb{V}^\vee$, toy model: $\text{End}(F(Y)) \times G$

When $E$ not polarized:

break symmetry to $T$ and enter the Weyl group.

Speculative explanation:

Consider $G \times E$, polarize that,

from Fukaya category

Limit $2\epsilon \to 0$: $G/T$ simplifies to sum of brane交

Turns out $G/T$ descends back where Weyl gp.
Theorem. For each quant rep $E$ of $G$, there is a space $E(G,E) \to$ Toda base, almost homogeneous for the Toda group scheme.

Moreover:
1. If $E$ is polarized, $E(G,E) \to$ Toda system.
2. If not, birational to a torus of rank $2$.
3. There is multiplication

$$E(G,E) \times E(G,E) \to E(G,E)$$

over the Toda base.

The $E(G,E)$ and $H^*(SL(G;E_E))$ by multiplication for $SL_G$.

(Polarized case: Braverman, Finkelberg, Nakajima
NP: different methods, Braverman et al.
An topological break of $\mathbb{C}$)

Concrete construction from Toda pover
+ superpotential $J$ GLSM for a
canonical TFT of $E$.

Polarized case:

Study $J$ functions in $E(G;0)$
Which remain regular by vertical translation
by a certain Lagrangian term
$\exp(c(1,1))$. 

NP case: Pass to Weyl cover of $E$

Choose a position half of $E$

Use GCDM superpotential to define a "natural" action of the Weyl gp

Descend back.

This gives functions in $E(G/E)$.

* : then are some mod 2 obstructions in NP case to be removed