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Non-Invertible Duality Defects

d = spacetime dim

Symmetry in QFT $\longleftrightarrow$ topological operators

$G^{(q)} \rightsquigarrow U_g(M^{d-q-1})$

$q = 0$ ordinary

$q = 1$ 1-form

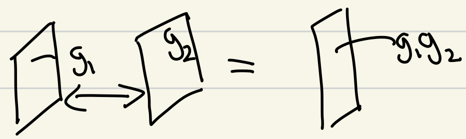
$g \in G$

Fusion algebra

group like

(invertible)

$U_{g_1} \times U_{g_2} = U_{g_1 g_2}$



Fusion of top operators can be more general (non-invertible)

$$U_i \times U_j = \sum_k U_k$$

more than one term

defects of different dimensions

Prominent Examples

- Verlinde lines in rational CFTs, e.g. ising fusion category

1 ; η ; σ

identity ; $\mathbb{Z}_2^{(0)}$; KW duality

$\eta \times \eta = 1$, $\sigma \times \sigma = 1 + \eta$

$\sigma \times \eta = \eta \times \sigma = \sigma$

• anyons in 3d TQFT $\sim$ non-invertible 1-form symmetry

<u>This talk</u> : construct non-invertible defect ("duality defect" $\mathcal{D}$)

in 3+1 d (works in general d) generalize KW

<u>Defect Construction</u>

(input duality)

$$\text{QFT } \mathcal{T} \cong \mathcal{T}/G^{(q)}$$

thy with $G^{(q)}$ sym — sym gauged

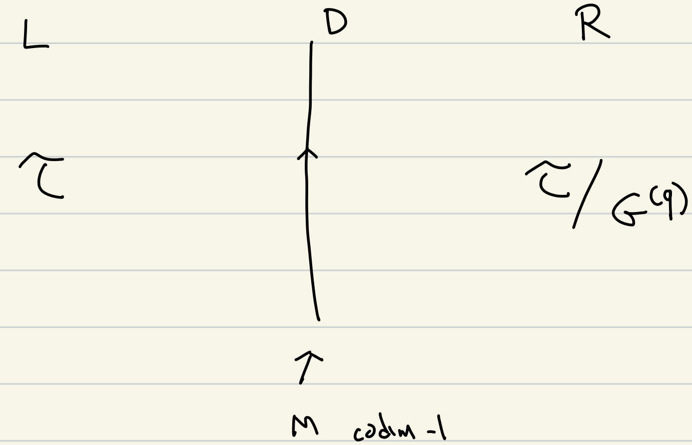
$\left(G^{(q)} = \mathbb{Z}_N^{(q)} \right.$ for simplicity$\left. \right)$

<u>Note</u> gauging $G^{(q)} \Longrightarrow$ new dual sym $G^{(d-q-2)}$ symmetry

only possible if $q = \frac{d-2}{2} \in \mathbb{Z}$

($q=0$ $d=2$; $\underline{\underline{q=1 \quad d=4}}$, ...)

Dirichlet Interface



$$S = \int_L \mathcal{L}_\tau + \int_R \mathcal{L}_{\tilde{\tau}}[a^{(q+1)}] + \cancelto{0}{SPT(a^{(q+1)})}$$

dynamical $G^{(q)}$ gauge fields

along interface locus M place dirichlet BC for $a^{(q+1)}$

$$a^{(q+1)}\Big|_M = 0$$

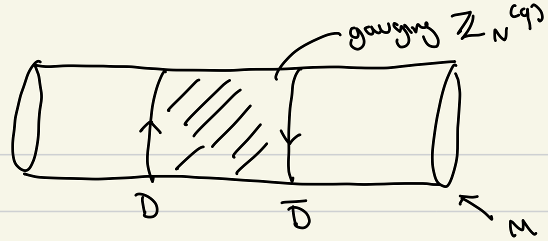
- topological interface $\left(da^{(q+1)} = 0 \right)$

duality assumption $\tau \cong \tau/G^{(q)} \Rightarrow$ topological defect in fixed theory

in R dual sym comes from ops $\eta(S) \equiv \exp\left(i \oint_S a^{(q+1)}\right)$

since $a^{(q+1)}\Big|_{M=0} \Rightarrow \eta \times D = D \times \eta = D$

$D\times\bar{D}$ Fusion Algebra



$M\times\mathbb{R}$

1+1 d $(M=S^1)$

Say $D\times\bar{D} \supseteq \underbrace{x}_{\text{unknown coeff}} \overset{\text{id}}{1} \implies D\times\bar{D} = x\underbrace{\sum_{i=0}^{N-1}\eta^i}_{\text{expected sum over Wilson lines in gauge thy}}$ $\qquad \eta = \exp\left(i\oint_{S^1} a^{(1)}\right)$

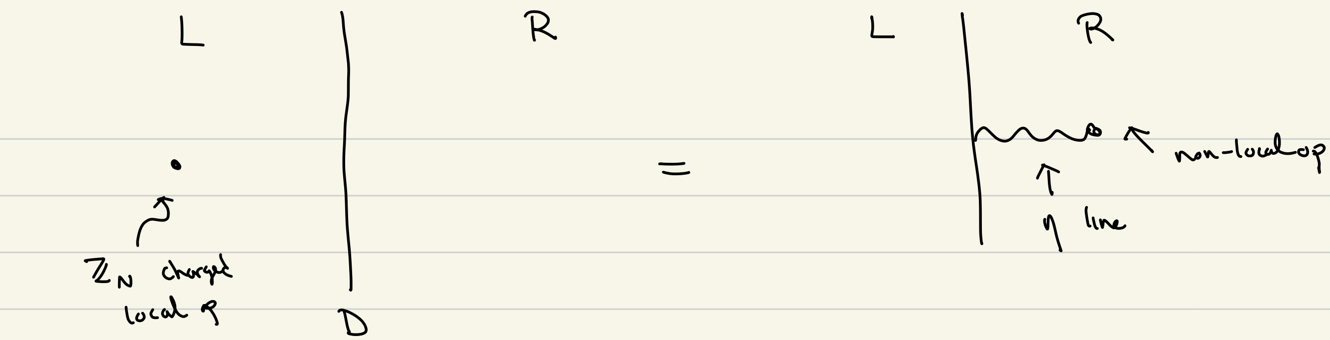
coefficient x comes from normalization of $\mathbb{Z}_N^{(0)}$ partition fnc in slab

$$x = \frac{1}{|H^0(S^1\times I, \partial(S^1\times I), \mathbb{Z}_N)|} = 1$$

relative cohomology b/c of dirichlet bc.

$G^{(q)} = \mathbb{Z}_N^{(0)}$: $\eta^i \quad i=0,\dots,N-1$, $\quad D\times\bar{D} = \sum_{i=0}^{N-1}\eta^i$ (Tambara-Yamagami fusion category)

Key property of D:



<u>3+1 D</u> | $\mathbb{Z}_N^{(1)}$ again $D \times \bar{D}$ by gauging $\mathbb{Z}_N^{(1)}$ in a slab $M \times I$ (some 3-manifold)

$$\left[\begin{matrix}\text{gauge fields to} \\ \text{sum}\end{matrix}\right] = H^2(M\times I, \partial(M\times I), \mathbb{Z}_N) \cong H_2(M, \mathbb{Z}_N)$$

$$D(M)\times \bar{D}(M) = x \sum_{S\in H_2(M,\mathbb{Z}_N)} \underbrace{\eta(S)}_{\mathbb{Z}_N^{(1)} \text{ sym defects}}$$

again a sum over sym defects

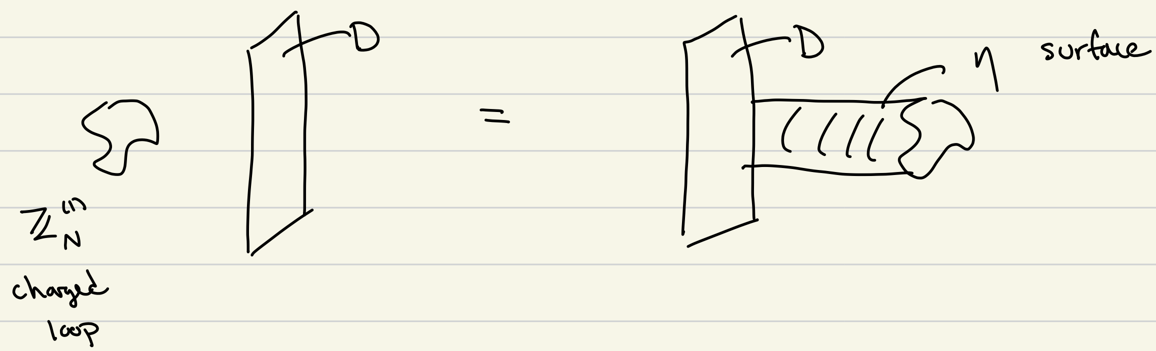
$$x = \begin{matrix}\text{normalization} \\ \text{of part func}\end{matrix} = \frac{|H^0(M\times I, \partial(M\times I), \mathbb{Z}_N)|}{|H^1(\ \cdot\ \cdot\ )|} = \frac{1}{N}$$

$(\mathbb{Z}_N^{(1)} = G^{(1)})$

$D(M)\times\eta = \eta\times D(M) = D(M)$, $D(M)\times\bar{D}(M) = \frac{1}{N}\sum_{S\in H_2(M,\mathbb{Z}_N)} \eta(S)$

$= H^1(M,\mathbb{Z}_N)$

Key property



Example U(1) Maxwell

defined by $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$, $U(1)_e^{(1)} \times U(1)_m^{(1)}$

gauge $\mathbb{Z}_N^{(1)} \subseteq U(1)_e^{(1)} \Rightarrow A \longrightarrow A/N \Rightarrow \tau \rightarrow \tau/N^2$

connection $U(1)$ $\qquad$ $U(1)/\mathbb{Z}_N \cong U(1)$

Use EM duality: $\tau \sim \frac{a\tau+b}{c\tau+d}$ $\quad a,b,c,d \in \mathbb{Z}$, $ad-bc=1$

eg S-dual $\quad -1/\tau = \tau/N^2 \Rightarrow \tau = iN \quad \left(\begin{matrix} \theta = 0 \\ g^2 = \frac{4\pi}{N} \end{matrix} \right)$

so at special τ we have duality defect D

$$S = \frac{N}{4\pi}\int_{x<0} dA_L \wedge * dA_L + \frac{N}{4\pi}\int_{x>0} dA_R \wedge * dA_R + \frac{iN}{2\pi}\int_{x=0} A_L \wedge dA_R \qquad \text{(Gaiotto-Witten)}$$

<u>EOM</u>

$$dA_L = -i * dA_R \underset{\substack{\uparrow \\ \text{EM at} \\ \tau = iN}}{=} \frac{1}{N}\, d\tilde{A}_R$$

<u>Further Examples</u>

- $SO(8)$ YM has $\mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(1)}$ duality defects

- $SU(N)$ $\mathcal{N}=4$ SYM at $\tau = i$ $\quad SU(N) \cong SU(N)/\mathbb{Z}_N$ $\quad$ gauging $\mathbb{Z}_N^{(1)}$

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- $PSU(N)$ YM at $\theta = \pi$

- spacetime $Y \times S^1 \times S^1$