Mathiness in Numbers

1 In the Beginning...

Definition 1.1. *An integer, a, is said to be even if and only if there is an integer n such that $a = 2n$.*

Definition 1.2. *An integer, b, is said to be odd if and only if there is an integer m such that $b = 2m + 1$.*

Notice this definition does not imply that all integers are either even or odd. Alternatively we could define odd numbers as being simply not even numbers, but then we would not have that odd number would have that form. For these problems assume that all integers are either odd or even.

1.1 Simply Even

If n be an even number then $n^2 + 1$ is an odd number. It is in fact also true that if $n^2 + 1$ is odd then n is even, but that is a proof that requires much much more.

1.2 Not So Simply Even

If $3n + 5$ is an odd number then $n^3 + 3$ is an odd number

1.3 Odd Couple

Let a and b be even integers and n and m be odd integers. Prove:

- $a + b$ is even
- $a + n$ is odd
- $n + m$ is even

1.4 Even Odder Couples

Let a and b be even integers and n and m be odd integers. Prove that:

- $a \cdot b$ is even
- $a \cdot n$ is even
- $n \cdot m$ is odd

1.5 The Number Theory of Combinations

Let $n \in \mathbb{Z}$, prove that either n or $n+1$ is an even number.

1.6 The Number Theory of Combinations II

Let $n \in \mathbb{Z}$, prove that n, $n+1$ or $n+2$ is a multiple of 3.

1.7 The Number Theory of Combinations III

Let $n \in \mathbb{Z}$, prove that $n(n+1)(n+2)$ is a multiple of 6.

1.8 Prime and Deadly

A natural number is prime if it only has two factors 1 and itself. For example, 5 is a prime number since it's only factors is 1 and 5. A non-example is 6, it has factors 1, 2, 3 and 6. Notice from the definition that 1 is not a prime number as it only has 1 factor. As a test qeustion consider the following problems.

- Find all primes of the form $3^n - 1$.
- Find all primes of the form $5^n - 1$.
- Find all primes of the form $5^n - 3^n$.

Note is that this is asking for you to find ALL primes of those forms, meaning you will have to prove that in fact you found them all.

1.9 Prime Triplet

Prove that $(3, 5, 7)$ is the only prime triplet of the form $(p, p+2, p+4)$.