Handout 2

August 6, 2015

Problem 1 Determine whether the equation is exact. If it is, then solve it.

$$(2x + y)dx + (x - 2y)dy = 0$$

Problem 2 Determine whether the equation is exact. If it is, then solve it.

$$(x/y)dy + (1 + \ln y)dx = 0$$

Problem 3 Determine whether the equation is exact. If it is, then solve it.

$$(y^2 + 2xy)dx - x^2 dy = 0$$

Problem 4 Find an integrating factor of the form $x^n y^m$ and solve the equation.

$$(2y^2 - 6xy)dx + (3xy - 4x^2)dy = 0$$

Definition The differential form $M(x,y)dx + N(x,y)dy$ is exact if there is a function $F(x,y)$ such that

$$\frac{\partial F}{\partial x} = M \qquad \text{and} \qquad \frac{\partial F}{\partial y} = N$$

If $Mdx + Ndy$ is exact, then the equation

$$M(x,y)dx + N(x,y)dy = 0$$

is called an exact equation.

Test for Exactness $M(x,y)dx + N(x,y)dy = 0$ is an exact equation if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Method for Solving Exact Equations

1. If $Mdx + Ndy = 0$ is exact, then $\frac{\partial F}{\partial x} = M$. Integrate this with respect to x to get
$$F(x,y) = \int M(x,y)dx + g(y)$$

2. To determine $g(y)$, take the partial derivative with respect to y of both sides and substitute N for $\frac{\partial F}{\partial y}$. Then solve for $g'(y)$.

3. Integrate $g'(y)$ to obtain $g(y)$ up to a numerical constant. Substitute for $g(y)$ to get $F(x,y)$.

4. The solution to $Mdx + Ndy = 0$ is given implicitly by $F(x,y) = C$.

Integrating Factor If $M(x,y)dx + N(x,y)dy = 0$ is not exact, but

$$\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$$

is exact, then $\mu(x,y)$ is called an integrating factor for the equation.