Handout 3

August 11, 2015

Find a general solution to the given differential equations.

1. $y'' - 5y' + 6y = 0$
2. $y'' + 6y' + 9y = 0$
3. $y'' + y = 0$
4. $y'' - y' - 2y = 0$

Homogeneous Second Order Equation with Constant Coefficients a, b, c constants with $a \neq 0$

$$ay'' + by' + cy = 0$$

Linearly Independent Solutions Two solutions y_1 and y_2 to the homogeneous equation on the interval I are said to be linearly independent on I if neither function is a constant times the other on I. This will be true provided their Wronskian

$$W[y_1, y_2](t) = y_1(t)y_2'(t) - y_1'(t)y_2(t)$$

General Solution to Homogeneous Equation: $c_1y_1 + c_2y_2$ If y_1 and y_2 are linearly independent solutions to the homogeneous equation, then a general solution is

$$y(t) = c_1y_1(t) + c_2y_2(t)$$

where c_1 and c_2 are arbitrary constants.

Form of General Solution The form of a general solution for a homogeneous equation with constant coefficients depends on the roots

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \qquad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

of the auxiliary equation $ar^2 + br + c = 0$, $a \neq 0$.

- When $b^2 - 4ac > 0$, the auxiliary equation has two distinct real roots r_1 and r_2 and a general solution is

$$y(t) = c_1e^{r_1t} + c_2e^{r_2t}$$

- When $b^2 - 4ac = 0$, the auxiliary equation has a repeated real root $r = r_1 = r_2$ and a general solution is

$$y(t) = c_1e^{rt} + c_2te^{rt}$$

- When $b^2 - 4ac < 0$, the auxiliary equation has complex conjugate roots $r = \alpha \pm i\beta$ and a general solution is

$$y(t) = c_1e^{\alpha t}\cos(\beta t) + c_2e^{\alpha t}\sin(\beta t)$$

Special Integrating Factors for Exact Equations Suppose the equation $M(x, y)dx + N(x, y)dy = 0$ is not exact.

An integrating factor of the equation is a function $\mu(x, y)$ such that the equation

$$\mu(x, y)M(x, y)dx + \mu(x, y)N(x, y)dy = 0$$

is exact.

If $(\partial M/\partial y - \partial N/\partial x)/N$ depends only on x, then

$$\mu(x) = \exp[\int (\frac{\partial M/\partial y - \partial N/\partial x}{N})dx]$$

is an integrating factor for the equation.

If $(\partial N/\partial x - \partial M/\partial y)/M$ depends only on y, then

$$\mu(y) = \exp[\int (\frac{\partial N/\partial x - \partial M/\partial y}{M})dy]$$

is an integrating factor for the equation.