Handout 4

August 13, 2015

Problem 1 Find a particular solution first by undetermined coefficients, and then by variation of parameters.

$$2y'' - 2y' - 4y = 2e^{2t}$$

Problem 2 Solve with variation of parameters.

$$y'' + y = \sec(t)$$

Problem 3 Solve with variation of parameters.

$$y'' + y = \tan^2(t)$$

Problem 4 Use the method of variation of parameters to show that

$$y(t) = c_1 \cos(t) + c_2 \sin(t) + \int_0^t f(s)\sin(t-s)ds$$

is a general solution to the differential equation

$$y'' + y = f(t)$$

where $f(t)$ is a continuous function on $(-\infty, \infty)$.

(Hint: Use the trigonometric identity $\sin(t-s) = \sin t \cos s - \sin s \cos t$)

Method of Variation of Parameters To determine a particular solution to $ay'' + by' + cy = f$:

1. Find two linearly independent solutions $\{y_1(t), y_2(t)\}$ to the corresponding homogeneous equation and take

$$y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$$

2. Determine $v_1(t)$ and $v_2(t)$ by solving the following system for $v_1'(t)$ and $v_2'(t)$ and integrating.

$$y_1 v_1' + y_2 v_2' = 0$$
$$y_1' v_1' + y_2' v_2' = \frac{f}{a}$$

3. Substitute $v_1(t)$ and $v_2(t)$ into the expression for $y_p(t)$ to obtain a particular solution.

Method of Undetermined Coefficients To find a particular solution to the differential equation

$$ay'' + by' + cy = Ct^m e^{rt}$$

where m is a nonnegative integer, use the form

$$y_p(t) = t^s(A_m t^m + ... + A_1 t + A_0)e^{rt}$$

with

- $s = 0$ if r is not a root of the associated auxiliary equation
- $s = 1$ if r is a simple root of the associated auxiliary equation
- $s = 2$ if r is a double root of the associated auxiliary equation

To find a particular solution to the differential equation

$$ay'' + by' + cy = Ct^m e^{\alpha t}\cos(\beta t)$$

or

$$ay'' + by' + cy = Ct^m e^{\alpha t}\sin(\beta t)$$

for $\beta \neq 0$, use the form

$$y_p(t) = t^s(A_m t^m + ... + A_1 t + A_0)e^{\alpha t}\cos(\beta t) + t^s(B_m t^m + ... + B_1 t + B_0)e^{\alpha t}\sin(\beta t)$$

with

- $s = 0$ if $\alpha + i\beta$ is not a root of the associated auxiliary equation
- $s = 1$ if $\alpha + i\beta$ is a root of the associated auxiliary equation.