Handout 5

August 18, 2015

Problem 1

1. Find the most general solution of the linear system.
2. Classify the stability of the critical point at $(0,0)$.

$$\frac{dx}{dt} = -3x - 5y$$
$$\frac{dy}{dt} = 2x - y$$

Stability of Linear Systems Assume the origin $(0,0)$ is an isolated critical point for the linear system

$$x'(t) = ax + by$$
$$y'(t) = cx + dy$$

where a, b, c, d are real and $ad - bc \neq 0$. Let r_1 and r_2 be the roots of the characteristic equation

$$r^2 - (a+d)r + (ad - bc) = 0$$

The stability of the origin and the classification of the origin as a critical point depends on the roots r_1 and r_2 as follows:

Roots	Type of Critical Point	Stability
distinct, positive	node	unstable
distinct, negative	node	asymptotically stable
opposite signs	saddle point	unstable
equal, positive	star or degenerate node	unstable
equal, negative	star or degenerate node	asymptotically stable
complex-value: positive real part	spiral point	unstable
complex-valued: negative real part	spiral point	asymptotically stable
complex-valued: pure imaginary	center	stable