Handout 6

August 25, 2015

Problem 1 Consider the system of differential equations:

$$x_1' = x_2 + e^t$$
$$x_2' = -2x_1 + 3x_2$$

1. Write the system in the matrix form:

$$x' = Ax + f$$

2. Solve the homogeneous system.

3. Solve the nonhomogeneous system using variation of parameters.

4. Write down the general solution to the system.

5. Check that your solutions satisfy the original system.

Problem 2 Show that two particular solutions to a nonhomogeneous system always differ by a solution to the corresponding homogeneous system.

Homogeneous Normal Systems

$$x'(t) = A(t)x(t)$$

where $A(t)$ is an $n \times n$ matrix, and $x(t)$ is an $n \times 1$ column vector.

Fundamental Matrix An $n \times n$ matrix $X(t)$ whose column vectors form a fundamental solution set for the homogeneous system is called a fundamental matrix.

General Solution to Homogeneous System If $X(t)$ is a fundamental matrix whose column vectors are $x_1(t), ..., x_n(t)$, then a general solution to the homogeneous system is

$$x(t) = X(t)c = c_1 x(t) + ... + c_n x_n(t)$$

where $c = \text{col}(c_1, ..., c_n)$ is an arbitrary constant vector.

Nonhomogeneous Normal Systems

$$x'(t) = A(t)x(t) + f(t)$$

where now we have added a vector function $f(t)$.

Variation of Parameters Let $X(t)$ be a fundamental matrix for the homogeneous system. A particular solution to the nonhomogeneous system is given by the variation of parameters formula

$$x_p(t) = X(t) \int X^{-1}(t) f(t) dt$$