Handout 8

September 1, 2015

Problem 1 Solve the IVP using the method of Laplace transforms.

$$y'' - 2y' + 5y = 0$$

$$y(0) = 2, y'(0) = 4$$

Problem 2 Solve the IVP using the method of Laplace transforms.

$$y' + 3y = 13\sin(2t)$$

$$y(0) = 6$$

Problem 3 Solve the IVP using the method of Laplace transforms.

$$y''' - y'' + y' - y = 0$$

$$y(0) = 1, y'(0) = 1, y''(0) = 3$$

Method of Laplace Transforms To solve an initial value problem:

1. Take the Laplace transform of both sides of the equation.
2. Use the properties of the Laplace transform and the initial conditions to obtain an equation for the Laplace transform of the solution and then solve this equation for the transform.
3. Determine the inverse Laplace transform of the solution by looking it up in a table or by using a suitable method (such as partial fractions) in combination with the table.

Laplace Transform The Laplace transform of a function $f(t)$ is the function $F(s)$ defined by

$$F(s) = \int_0^\infty e^{-st} f(t)dt$$

Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}(f)(s)$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$

Properties of Laplace Transform

- $\mathcal{L}(f+g) = \mathcal{L}(f) + \mathcal{L}(g)$
- $\mathcal{L}(cf) = c\mathcal{L}(f)$ for any constant c
- $\mathcal{L}(e^{at}f(t)(s) = \mathcal{L}(f)(s-a)$
- $\mathcal{L}(f')(s) = s\mathcal{L}(f)(s) - f(0)$
- $\mathcal{L}(f'')(s) = s^2\mathcal{L}(f)(s) - sf(0) - f'(0)$
- $\mathcal{L}(f^{(n)})(s) = s^n\mathcal{L}(f)(s) - s^{n-1}f(0) - s^{n-2}f'(0) - ... - f^{(n-1)}(0)$
- $\mathcal{L}(t^n f(t))(s) = (-1)^n \frac{d^n}{ds^n}(\mathcal{L}(f)(s))$