Complex Variables II: Homework 1

Start reading Chapter 10 in Stewart and Tall.

I. Exercises 10: #3 (ii) and (iv); #8; #11; #14

(Note: For #3, you could compute $f^{(n)}(0)$ for all n to find the Taylor series. Remember, though, if you find any power series such that $f(z) = \sum a_n z^n$, the a_n's must equal $f^{(n)}(0)/n!$ – therefore, you might want to look for simpler ways of finding these series.)

II.:

1. Prove that $F(z) = \dfrac{1}{2\pi i} \displaystyle\int_{C_1(0)} \frac{\overline{w}}{w-z}\, dw = 0$ for every $z \in N_1(0)$.

2. On what domain is $\sqrt{z}$ single-valued and analytic? Find the Taylor series expansion around $z = 1$ and compute its radius of convergence.

3. Let $p(z) = a_o + a_1 z + ... + a_n z^n$, where $a_n \neq 0$. Prove that there exists an $R > 0$ such that $|p(z)| \geq |a_n||z|^n/2$ for all $|z| \geq R$.

4. Prove or disprove: If f is analytic on $N_1(0)$ and f^2 is a polynomial, then f is a polynomial.