Complex Variables II: Homework 7

Read Chapter 13 in Stewart and Tall.

I. Exercises 13: #3, #4, #11

II.:
1. (a) Let D be any simply connected domain with $D \neq \mathbb{C}$. Prove that there is no conformal mapping from $\mathbb{C}$ onto D.

(b) Find an analytic mapping from $N_1(0)$ *onto* $\mathbb{C}$.

2. Prove that there is no conformal map from the deleted disc $\{z \in \mathbb{C} : 0 < |z| < 1\}$ onto an annulus $\{z \in \mathbb{C} : r < |z| < R\}$ (where $r > 0$). (Hint: Assume f is such a function – what can you prove about the point 0?)

3. (a) Let D be a simply connected domain with $D \neq \mathbb{C}$. Assume that f is a conformal map from D onto the unit disc $N_1(0)$; also fix a point $z_o \in D$. Find a conformal map g from D onto the unit disc $N_1(0)$ such that $g(z_o) = 0$ and $g'(z_o) > 0$.

(b) Let D be *any* simply connected domain. Let $z_1, z_2 \in D$. Prove that there is a conformal map f from D onto itself such that $f(z_1) = z_2$.

4. Let D be a simply connected domain ($D \neq \mathbb{C}$), and let $z_o \in D$. Consider the set

$$\mathcal{G} = \{g : D \to N_1(0) : g \text{ is analytic }, g'(z_0) > 0\}.$$

(a) Prove that $\sup_{g \in \mathcal{G}} g'(z_o) = M < \infty$.

(b) Let $f : R \to N_1(0)$ be an analytic function such that $f'(z_o) = M$. Prove that f is one to one. (Hint: prove that f must actually be the unique conformal map (from D to $\mathbb{C}$, with z_o mapped to 0) defined by the Riemann mapping theorem.)