

Complex Variables II: Homework 8

Read Chapter 14 in Stewart and Tall.

I. Exercises 14: #2, #5, #7, #9, #20

II.:

1. Prove that $\prod_{k=0}^{\infty} (1 + z^{2^k})$ converges uniformly to $\frac{1}{1-z}$ in $|z| < 1$.

2. Define a function that is analytic in $|z| < 1$ such that $f(z) = 0$ if and only if $z = 1 - \frac{1}{k}$ for all $k = 1, 2, 3, \dots$

3. Prove that

$$\cos(\pi z) = \prod_{k=0}^{\infty} \left(1 - \frac{4z^2}{(2k+1)^2} \right)$$

4. Let $f(z)$ be a function with genus 0 or 1 and assume that $f(z)$ has only real zeros and also that $f(z)$ is real on the real axis. Prove that all the zeros of $f'(z)$ are real. Hint: Consider $\operatorname{Im} \left(\frac{f'(z)}{f(z)} \right)$.