Math CS-120: Homework 3

Read Chapter 4 in Stewart and Tall.

I.

1. Prove that $f(z)$ is analytic on a domain S if and only if $g(z) = \overline{f(\bar{z})}$ is analytic on $T = \{z : \bar{z} \in S\}$.

2. Using the definitions of $\partial f/\partial z$ and $\partial f/\partial \bar{z}$ from class, show that if $u(x, y)$ is a harmonic function, then

$$\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$$

3. (a) If $u(x, y) = x^2 - y^2$ find all its conjugate harmonic functions $v(x, y)$. What are the analytic functions $f = u + iv$ (in terms of z)?
(b) Veryify that $u(x, y) = \log(\sqrt{x^2 + y^2})$ is harmonic everywhere in $\mathbb{C} \setminus \{0\}$. Find a function $f(z) = u(x, y) + iv(x, y)$ that is analytic in the right half plane $\{z : \text{Re } z > 0\}$.

II. Exercises 4: #4, #5, #7, #8, #12, #13