

Review Problems for the Final

Math 3A, Fall 2005

The final will cover Chapters 1.5-1.6; 2.1-3,5-9; 3.1-3,5-8,10; 4.1-5,7. The final is on Tuesday, Dec. 6th, from 7:30-10:30pm in GIRV 1004.

Review of Chapters 1-3

For further practice, please see the midterm reviews and go over your old tests, quizzes, and homework. You may also do problems from the corresponding sections and chapter reviews in your book.

1. Use the definition of derivative to find the derivative of the function $f(x) = \sqrt{1+x}$ at the point $a = 1$.

2. Find the derivatives of the following functions.

(a) $f(x) = \frac{x^2}{1+e^x}$

(b) $y = \arctan(\sqrt{x^2-1})$

(c) $h(t) = t \cos t^2$

(d) $f(x) = \frac{(x^3-1)^4}{(2x+1)(3x^2+1)^5}$

(e) $y = (\sin x)^x$

3. Find f' in terms of g' .

(a) $f(x) = g(e^x)$, (b) $f(x) = e^{g(x)}$, (c) $f(x) = g(x^2)$, (d) $f(x) = x^2g(\sin x)$

4. Consider the curve $xe^y = y - 1$. Show that the point $(0, 1)$ is on the curve, and then find the tangent line to the curve at the point $(0,1)$.

5. Two cars start moving from the same point. One travels north at 60 mi/h, and the other travels west at 25 mi/h. At what rate is the distance between the two cars increasing two hours later?

Review of Chapter 4

1. Find the absolute minimum and absolute maximum values of the function $f(x) = e^x - e^{2x}$ on the interval $[-1, 1]$.
2. Find the absolute maximum and absolute minimum as well as all local maxima and minima on the interval $[0, 7]$ for the function $f(x) = (x^3 - 11x^2 + 24x)^2$.
3. Show that $\sqrt{1+x} < 1 + \frac{1}{2}x$ if $x > 0$. (Hint: Apply the Mean Value Theorem to the function $f(x) = \sqrt{1+x}$.)
4. Evaluate the following limits.
 - (a) $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$
 - (b) $\lim_{x \rightarrow 0^+} x^2 \ln x$
 - (c) $\lim_{x \rightarrow 0^+} (e^x - 1)^x$
5. Find all of the inflection points of the function $f(x) = ax^4 + bx^3 + cx^2 + d$. (Note: Your answer may depend on the values of the constants a, b, c , and d .)
6. For the following functions, find A. the domain, B. the x- and y-intercepts, C. any symmetry, D. all asymptotes, E. the intervals on which f is increasing and decreasing, F. all local maxima and minima, and G. the intervals on which f is concave up and concave down and all inflection points. Finally, using this information, sketch a graph of the function.
 - (a) $f(x) = x + \sqrt{1-x}$
 - (b) $f(x) = \frac{1}{1-x^2}$
7. Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.
8. A metal storage tank of fixed volume V is to be constructed in the shape of a circular cylinder, with height h and radius r , topped by a hemisphere of radius r . What dimensions will require the least amount of material?