Answers to Review Problems for the Second Midterm

Math 3A, Fall 2005

Review for Chapters 1-3

1. $f'(1) = \lim_{h\to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h} = \dfrac{1}{2\sqrt{2}}$

2. (a) $f'(x) = \dfrac{2x(1+e^x) - x^2e^x}{(1+e^x)^2}$

(b) $y' = \dfrac{1}{x\sqrt{x^2-1}}$

(c) $h'(t) = \cos t^2 - 2t^2 \sin t^2$

(d) $f'(x) = \left(\dfrac{(x^3-1)^4}{(2x+1)(3x^2+1)^5}\right)\left(\dfrac{12x^2}{x^3-1} - \dfrac{2}{2x+1} - \dfrac{30x}{3x^2+1}\right)$

(e) $y' = (\sin x)^x(\ln \sin x + x \cot x)$

3. (a) $f'(x) = e^x g'(e^x)$

(b) $f'(x) = e^{g(x)}g'(x)$

(c) $f'(x) = 2xg'(x^2)$

(d) $f'(x) = 2xg(\sin x) + x^2 \cos x g'(\sin x)$

4. $0e^1 = 1 - 1$; $y = ex + 1$

5. 65 mi/h

Review for Chapter 4

1. absolute minimum: $f(1) = e(1-e)$, absolute maximum: $f(\ln \frac{1}{2}) = \frac{1}{4}$

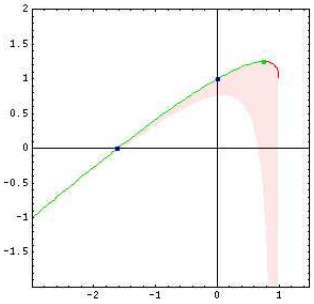
2. absolute minimum: $f(0) = f(3) = 0$, absolute maximum: $f(6) = 6^4$, local minima: $f(0) = 0$ and $f(3) = 0$,
local maxima: $f(\frac{4}{3}) = \frac{20^4}{3^6}$ and $f(6) = 6^4$.

3. By the MVT, for some $c \in (0, x)$, $\sqrt{1+x} - 1 = \frac{1}{2\sqrt{1+c}} < \frac{1}{2}$.

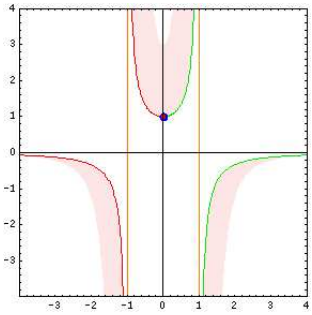
4. (a) 14, (b) 0, (c) 1

5. If $36b^2-96ac > 0$, then there are two inflection points with x-coordinates $\dfrac{-6b \pm \sqrt{36b^2 - 96ac}}{24a}$.
If $36b^2 - 96ac \leq 0$, then there are no inflection points.

6(a) A. $(-\infty, 1]$
B. y-intercept: $(0, 1)$; x-intercept $(\frac{-1-\sqrt{5}}{2}, 0)$
C. none; D. none
E. increasing on $(-\infty, \frac{3}{4})$ and decreasing on $(\frac{3}{4}, 1)$.
F. a local maxima occurs at $x = \frac{3}{4}$
G. concave down on $(-\infty, 1]$.



(b) A. all $x \neq \pm 1$ (or $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$)
B. y-intercept:(0,1); no x-intercepts; C. even
D. vertical asymptotes: $x = 1$, $x = -1$; horizontal asymptotes $y = 0$
E. decreasing on $(-\infty, -1)$ and $(-1, 0)$ and increasing on $(0, 1)$ and $(1, \infty)$
F. a local minima occurs at $x = 0$
G. concave down on $(-\infty, -1)$ and $(1, \infty)$ and concave up on $(-1, 1)$



7. $\left(\dfrac{45}{37}, \dfrac{63}{37}\right)$

8. $r = \sqrt[3]{\dfrac{3V}{5\pi}}; h = \sqrt[3]{\dfrac{3V}{5\pi}}$