Review Problems for the First Midterm

Math 3A, Fall 2005

1. Solve the following equations for x.
(a) $\ln x = -5$
(b) $e^{e^x} = 2$
(c) $\ln(2x) + \ln(x - \frac{1}{2}) = 0$

2. Find the inverses of the following functions. Express your final answer for f^{-1} as a function of x. What are the domain and range of each function and its inverse?
(a) $f(x) = \frac{x+1}{3x}$
(b) $f(x) = e^{x^3-1}$

3. Compute the following limits.
(a) $\lim_{x\to 1} \dfrac{\sqrt{x-1}-1}{x-2}$
(b) $\lim_{h\to 0} \dfrac{\frac{1}{h+2} - \frac{1}{2}}{h}$
(c) $\lim_{x\to -\infty} \dfrac{\sqrt{9x^2-1}}{3x}$
(d) $\lim_{x\to 0^+} \dfrac{x^{\frac{1}{3}}}{x^2+x}$
(e) $\lim_{x\to -2^-} \dfrac{x^2}{x+2}$

4. Prove that that the limit of $f(x) = x^2\cos(\frac{1}{x})$ as x approaches 0 exists. What is the limit? State the names of any theorems you use.

5. Find a number c such that the following function is continuous every-

where on $(-\infty, \infty)$. Justify your answer.

$$f(x) = \begin{cases} c - x^2 & \text{if } x \leq 0 \\ \\ 3x^2 - cx & \text{if } x > 0 \end{cases}$$

6. Use the Intermediate Value Theorem to show that the following equation has at least one negative root.

$$2x^3 + x^2 + 2 = 0$$

7. Find the domain and all horizontal asymptotes, vertical asymptotes, and x- and y-intercepts of the function f given below. Show all your work. Use this information to draw a rough sketch of the graph of f.

$$f(x) = \frac{(x-2)^2}{x^2 + 3x - 10}.$$

8. (a) Find the equation of the tangent line to the curve $y = 9 - 2x^2$ at the point $(2, 1)$.
(b) Find the equation of the tangent line to the function $f(x) = \frac{1}{x}$ at the point $x = 1$.

9. Let $f(x) = \frac{x^2-1}{x^2+1}$. We can compute that $\lim_{x\to\infty} f(x) = 1$. By the definition of infinite limits, this means that we may make the values of $f(x)$ arbitrarily close to 1 for sufficiently large x. In order to make f be within $\frac{1}{100}$ of 1, how large must you take x?

In other words, find a number N such that $|f(x) - 1| < \frac{1}{100}$ whenever $x > N$.