Review Problems for the Second Midterm

Math 3A, Fall 2005

1. Express the following limit as a derivative and find its value.

$$\lim_{h\to 0} \frac{(3+h)^4 - 81}{h}$$

2. (a) Find the equation of the tangent to the curve $y = e^x$ that is parallel to the line $4x - 2y = 3$.
(b) Find the equation of the tangent line to the curve $y = e^x$ that passes through the origin.

3. Differentiate
(a) $f(x) = \sqrt[3]{x}\left(\dfrac{1}{\sqrt[3]{x^4}} - \sqrt{x}\right)$
(b) $y = (1 + x^2)\arctan x$
(c) $s(t) = \sqrt{\cos\sqrt{t}}$
(d) $f(x) = \log_a(\pi x^2)$
(e) $y = \dfrac{\ln(x^2 e^x)}{x}$

4. Consider the curve defined by the equation $x^2 + y^2 = (3x^2 + 3y^2 - x)^2$.
(a) Find y'.
(b) Find the tangent line to the curve at the point $(0, -\frac{1}{3})$.
(c) Find y''. Express your final answer in terms of only x and y.

5. Use logarithmic differentiation to find y'. Express your answer as a function of x.
(a) $y = \dfrac{x^{3/4}\tan(3x)}{(x+2)^5 \ln x}$
(b) $y = x^{\sin(e^x)}$

6. A particle moves on a vertical line so that its coordinate at time t is $y = 2t^3 - 7t^2 + 4t + 1$, $t \geq 0$.
(a) Find the velocity and acceleration of the particle as functions of t.
(b) When is the particle moving upward and when is it moving downward?
(c) Find the total distance that the particle travels in the time interval $1 \leq t \leq 3$.
(d) When is the particle speeding up and when is it slowing down?

7. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep? (The volume of a cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.)

8. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?