Math 117: Quantifiers and Negation

$\forall$ means __________________________ $\exists$ means __________________________

Rewrite each of the following statements using $\forall$, $\exists$ and $\ni$. Then, prove or disprove the statement, giving an example or counterexample where appropriate:

For all $x \in \mathbb{R}$, $x^2 > 0$.

<u>There exists</u> $x \in \mathbb{R}$ <u>such that</u> $x^2 > 0$.

For all $x \in \mathbb{R}$ such that $|x - 5| < 2$, $x^2 + 30 < 13x$.

For every $y \in \mathbb{R}$, <u>there exists</u> $x \in \mathbb{R}$ <u>such that</u> $x + y = 0$.

$\star$ For every $n \in \mathbb{N}$, $F(n) = 2^{2^n} + 1$ is prime.

Negate each of the following statements (assume x, y and z are real numbers):

$\forall x, x^2 > 0$

$\forall x \ni |x - 5| < 2, x^2 + 30 < 13x$

$\exists x \ni x < 0 \ \wedge \ x^4 - 5 = 0$

$\forall y, \exists x \ni x + y = 0.$

If $x^2 \geq 3$ or $y^7 \geq 2$, then $z \geq x^2 + y^2$ implies that $z > 1$.