Math 117: Sequences

A **sequence** is a function whose domain is _____.

So, if s : _____ $\to \mathbb{R}$ is a sequence of real numbers, for each ____________, $s($___$)$ must return a unique real number. (So, the *range* of a sequence of real numbers is $\mathbb{R}$.)

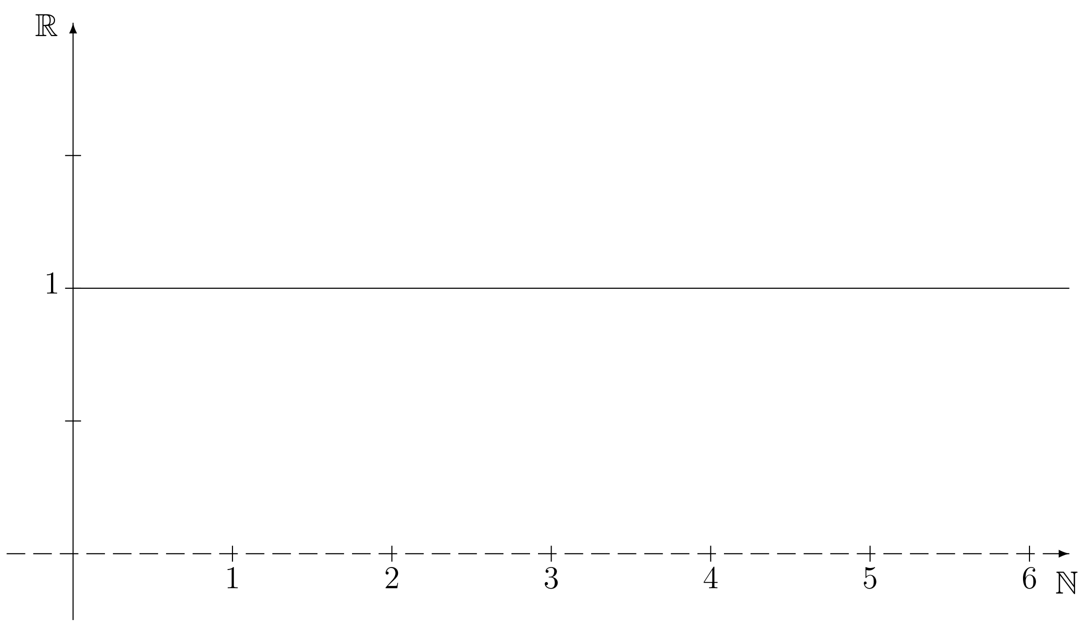
Commonly, we write ______ instead of ______ for the ______ term in a sequence. To refer to the entire sequence, use parentheses: $(s_1, s_2, s_3, ...)$ or (____) or (____)____.

Example 1. $\left((-1)^n\right)$ is the sequence $(-1, 1, -1, 1, -1, 1, ...)$.

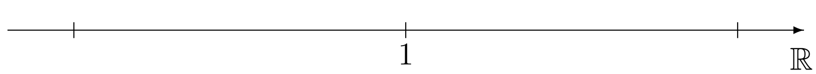
Example 2. Consider the sequence (s_n) where s_n is given by $1 + \frac{(-1)^{n+1}}{n}$.

$$(s_n) = \left(\; 2 \;,\; \tfrac{1}{2} \;,\; \underline{\hspace{2cm}}, \underline{\hspace{2cm}}, \underline{\hspace{2cm}}, \underline{\hspace{2cm}}, \;\; ... \; \right)$$

Thinking of the sequence (s_n) as a function, what does the graph look like?



Thinking of all the terms in the sequence (s_n) as a *subset* of real numbers $\{s_1, s_2, ...\} = \{s_n : n \in \mathbb{N}\}$, we draw this set on a real number line:



This sequence *converges* to the real number 1.

Definition. A sequence (s_n) of real numbers is said to **converge** to the real number s iff

______________________________, there exists _____________________________________

such that ____________________________________, $|s_n - s| <$ ______.

Notation. If a sequence (s_n) converges to s, s is called the **limit** of the sequence (s_n) and we write $\lim_{n\to\infty} s_n = s$. or $s_n \to s$.

Question: Why can we say "s is *the* limit of (s_n)"?

Theorem. If a sequence converges, the limit of the sequence is unique.

Proof. Let (s_n) be a sequence that converges. Let s be a limit of (s_n) and let t be a limit of (s_n). Letting $\epsilon > 0$ be given, there must exist _________________ such that

____

and there must exist _________________ such that

____

There exists $n_o \in \mathbb{N}$ such that ______________________________.

Therefore, by the ______________________________,

$$|s - t| = |(s - \underline{\qquad\quad}) + (\underline{\qquad\quad} - t)|$$

$$\leq \underline{\qquad\qquad\qquad\qquad\qquad}$$

$$< \underline{\qquad\qquad\qquad\qquad\qquad}$$

We have shown that for every $\epsilon > 0$, $|s - t| < \epsilon$. This implies that $|s - t| =$ ______.

Therefore, __________________ $\square$