A **sequence** is a function whose domain is \( \mathbb{N} \).

So, if \( s : \mathbb{N} \rightarrow \mathbb{R} \) is a sequence of real numbers, for each \( n \in \mathbb{N} \), \( s(n) \) must return a unique real number. (So, the **range** of a sequence of real numbers is \( \mathbb{R} \).)

Commonly, we write \( s_n \) instead of \( s(n) \) for the \( n \)th term in a sequence. To refer to the entire sequence, use parentheses: \((s_1, s_2, s_3, \ldots)\) or \((\ldots)\) or \((\ldots)\).

**Example 1.** \( (-1)^n \) is the sequence \((-1, 1, -1, 1, -1, 1, \ldots)\).

**Example 2.** Consider the sequence \((s_n)\) where \( s_n \) is given by \( 1 + \frac{(-1)^{n+1}}{n} \).

\[
(s_n) = \left( 2, \frac{1}{2}, \ldots, \ldots, \ldots, \ldots, \ldots \right)
\]

Thinking of the sequence \((s_n)\) as a function, what does the graph look like?

Thinking of all the terms in the sequence \((s_n)\) as a **subset** of real numbers \( \{s_1, s_2, \ldots\} = \{s_n : n \in \mathbb{N}\} \), we draw this set on a real number line:

This sequence **converges** to the real number 1.
**Definition.** A sequence \((s_n)\) of real numbers is said to converge to the real number \(s\) iff 

\[
\lim_{n \to \infty} s_n = s
\]

such that \(|s_n - s| < \epsilon\).

**Notation.** If a sequence \((s_n)\) converges to \(s\), \(s\) is called the limit of the sequence \((s_n)\) and we write \(\lim_{n \to \infty} s_n = s\) or \(s_n \to s\).

**Question:** Why can we say “\(s\) is the limit of \((s_n)\)”?

**Theorem.** If a sequence converges, the limit of the sequence is unique.

**Proof.** Let \((s_n)\) be a sequence that converges. Let \(s\) be a limit of \((s_n)\) and let \(t\) be a limit of \((s_n)\). Letting \(\epsilon > 0\) be given, there must exist \(n_0 \in \mathbb{N}\) such that

\[
|s_n - s| < \epsilon.
\]

Therefore, by the triangle inequality,

\[
|s - t| = |(s - s_n) + (s_n - t)|
\]

\[
\leq \epsilon + \epsilon
\]

\[
< 2\epsilon
\]

We have shown that for every \(\epsilon > 0\), \(|s - t| < \epsilon\). This implies that \(|s - t| = 0\). Therefore, \(\square\).