Midterm Review

Math 124A: PDEs

§1.2 First-order, linear equations

- Constant coefficient equations
  - Be able solve the equation $au_x + bu_y = 0$ using the method of characteristics.
  - Also understand how to find the general solution using a change of coordinates.
  - Find solutions of a first-order PDE with a given condition. Example: Solve $2u_x - u_y = 0$ with the condition $u(x, x) = x^2$. Graph the characteristic lines for this PDE.
  - Example: Solve the non-homogeneous equation $u_x+2u_y = x+7y$ using the change of coordinates method.
  - Example: Solve the equation $u_x + 2u_y + 5u = x + 7y$.
- Variable coefficient equations
  - Know how to find the characteristic curves for a homogeneous equation and determine the general form of the solution.
  - Example: Find the characteristic curves of the PDE $(1 + x^2)u_x + 2xyu_y = 0$. What is the general solution of this PDE?

§2.3, §2.4, §3.1, §3.3 Diffusion Equation

- Be able to state the maximum principle. (Also, see §2.3, Exercise 1.)
- Understand the energy method: Multiply $u_t = ku_{xx}$ by u and integrate. (See §2.3, page 43 and Exercise 8.)
- Know that $S(x, t)$, the source function, solves (DE) with initial data $S(x, 0) = \delta(x)$.
- Know the formula for the solution $u(x, t)$ of (DE) with initial condition $u(x, 0) = \phi(x)$.
- Know how to solve the diffusion equation on the half-line with initial condition $u(x, 0) = \phi(x)$ and either the Dirichlet boundary condition $u(0, t) = 0$ the Neumann boundary condition $u_x(0, t) = 0$. (E.g., §3.1 Exercise 1 and Homework 5, #4.)
- The diffusion equation with a source:
  - Understand the formula for the solution. (Go over Homework 5, #5.)
  - Example: Solve

$$\begin{cases} u_t - u_{xx} = \sinh(x) & \text{on } -\infty < x < \infty \\ u(x, 0) = x^2 \end{cases}$$

  Use the fact that the solution of the homogeneous diffusion equation ($k = 1$) with initial value $\sinh(x)$ is given by $e^t \sinh(x)$.