Math 5B: Final Review Problems

Here are some review problems for the final. You may also want to look over problems from your homeworks, midterm and quizzes, and the midterm review. Good luck!

Some useful formulas:

The area of the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ is πab.

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

1. Review limits, partial derivatives (including the chain rule and implicit differentiation), tangent planes, finding global maxima and minima, and the second derivative test.

2. Let $\mathbf{F} = (6x^2y + x - 1)\mathbf{i} + (z\sec^2(y) + 2x^3)\mathbf{j} + \tan(y)\mathbf{k}$. Verify that curl $\mathbf{F} = 0$. Find all functions f such that $\mathbf{F} = \nabla f$.

3. Consider the surface S given by the parameterization $\mathbf{r}(u, v) = (u\cos v, u\sin v, v)$, for $0 \le u \le 1$ and $0 \le v \le 2\pi$. If $f(x, y, z) = z^2e^x$, set up (but do not evaluate!) the integral you would compute to find $\iint_S f\, dS$. What integral would you want to evaluate to find the surface area of S?

4. (a) From the book: Exercise # 23 in Section 6.4.
(b) Consider the change of variables

$$\begin{aligned} u &= xy \\ v &= x^2 - y^2 \end{aligned}$$

What region is the square in the xy-plane $0 \le x \le 1$ and $0 \le y \le 1$ mapped to in the uv-plane? (Hint: Where is each line on the boundary of the square – $x = 0$, $x = 1$, $y = 0$, and $y = 1$ – mapped to in the uv-plane?) Evaluate the following integral using this change of variables:

$$\iint_D xy(x^2 + y^2)\sqrt[3]{x^2 + x^2y^2 - y^2}\, dx\, dy$$

where D is the square $0 \le x \le 1$ and $0 \le y \le 1$.

5. Evaluate

$$\iint_R \frac{y}{x} e^{x^2+y^2}\, dx\, dy,$$

where R is the region bounded by the lines $y = x$, $y = 0$ and the half-circle $x = \sqrt{1 - y^2}$.

6. (a) Evaluate $\int_0^1 \int_1^y \frac{1}{1+y^2}\, dx\, dy$.

(b) Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$. (Hint: There's no way to find an antiderivative for $\frac{\sin x}{x}$!)

7. Find the volume of the region inside the cylinder $x^2 + y^2 = 1$ and the sphere of radius 2 centered at the origin $(0, 0, 0)$.

8. Find the area of a region bounded by the lines $x = -\frac{1}{\sqrt{2}}$, $x = \frac{1}{\sqrt{2}}$, and the circle of radius 1 centered at $(0, 0)$.

9. Set up the integral (but do not evaluate!) that will give you the length of the curve given by the following parametric equations: For $0 \le t \le \pi$,

$$\begin{aligned} x &= 3\cos(t) + 5 \\ y &= \sin(3t) \end{aligned}$$

10. Consider the curve parameterized by $\mathbf{c}(t) = (t\cos(t), t\sin(t), t)$.
(a) Show that this curve lies on the surface $x^2 + y^2 = z^2$.
(b) Find a parametric equation for the tangent line to the curve at the point $(0, \frac{\pi}{2}, \frac{\pi}{2})$.

11. Verify Green's theorem by computing the following integral in two different ways (once by doing the path integral and once by doing a double integral):

$$\oint_{\mathbf{c}} ye^x dx + xdy$$

where **c** is the counter-clockwise curve around the triangle with vertices (0,0), (3,0), and (1,2).

12. Evaluate the following path integrals:

(a) $\oint_{\mathbf{c}} xy^6 dx + (3x^2y^5 + 6x)\, dy$ where **c** is the ellipse $x^2 + 4y^2 = 4$.

(b) $\int_{\mathbf{c}} (2xe^y - 1)\, dx + x^2e^y\, dy$ where **c** is the parabola $y = 2(x-1)^2$ from $(1, 0)$ to $(2, 2)$.

(c) $\int_{\mathbf{c}} x^2y\, dx + xy^2\, dy$ where **c** is the curve $y = e^x$ from $(0, 1)$ to $(2, e^2)$.

(d) $\oint_{\mathbf{c}} x^2\, ds$ where **c** is the circle of radius 2 centered at $(1, 0)$.