Math 5B: Final Review Answers

2. $f(x,y) = 2x^3y + \frac{x^2}{2} - x + z\tan(y) + C$.

3. $\iint_S f\,dS = \int_0^{2\pi}\int_0^1 v^2 e^{u\cos(v)}\sqrt{u^2+1}\,du\,dv$

Surface area of $S = \iint_S dS = \int_0^{2\pi}\int_0^1 \|\mathbf{N}(u,v)\|\,du\,dv = \int_0^{2\pi}\int_0^1 \sqrt{u^2+1}\,du\,dv$

4.(a) *Hint: To find the domain, first solve for u and v. Along the curve $y = 1/x$, how are u and v related to one another? Look at $v^2 - u^2$!* Answer: $\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\sqrt{1+u^2}}^{\sqrt{1+u^2}} 8uv\,dv\,du = 0$.

(b) $\iint_D xy(x^2+y^2)\sqrt[3]{x^2+x^2y^2-y^2}\,dx\,dy = \int_0^1\int_{u^2-1}^{1-u^2}\frac{1}{2}u\sqrt[3]{u^2+v}\,dv\,du = \frac{3}{28}$

5. $\frac{(e-1)\ln(\sqrt{2})}{2}$

6.(a) $\int_0^1\int_1^y \frac{1}{1+y^2}\,dx\,dy = \frac{1}{2}\ln(2) - \frac{\pi}{4}$

(b) $\int_0^1\int_y^1 \frac{\sin x}{x}\,dx\,dy = 1 - \cos(1)$

7. Volume $= 4\pi\left(\frac{8}{3} - \sqrt{3}\right)$

8. Area $= 1 + \frac{\pi}{2}$

9. $\int_0^{\pi} 3\sqrt{\sin^2(t) + \cos^2(3t)}\,dt$

10. (a) Every point $(x(t), y(t), z(t))$ on the curve satisfies the equation $(x(t))^2 + (y(t))^2 = (z(t))^2$: i.e., $(t\cos(t))^2 + (t\sin(t))^2 = (t)^2$ is true for every t.

(b) $\mathbf{l}(u) = (-\frac{\pi}{2}u, \frac{\pi}{2} + u, \frac{\pi}{2} + u)$

11. $\oint_{\mathbf{c}} ye^x\,dx + x\,dy = \iint_D (1 - e^x)\,dx\,dy = 1 + 3e - e^3$.

12. (a) 12π; (b) $4e^2 - 2$; (c) $2e^2 + \frac{5}{9}e^6 - \frac{17}{9}$; (d) 12π