Math 5B: Midterm Review Problems

Also review all your homeworks; if there are problems or concepts that you are still unsure about, please come to office hours sometime next week! The midterm may cover any of the topics we've discussed from Chapters 1, 2, and 4.

1. Find the value of the limit if it exists. Otherwise, prove it doesn't exist.
   (a) $\lim_{(x,y)\to(0,2)} e^{-\frac{x-1}{y^2}}$
   (b) $\lim_{(x,y)\to(1,0)} \frac{xy(y-x+1)}{(x-1)^2+y^2}$

2. Let $f(x,y) = x^2 + 4y^2$. Draw some of the level curves of the surface (for example, the curves corresponding to $c = 0, c = 1, c = 4, c = 9$). Find the gradient of f, ∇f. Draw the gradient vectors of f at the points $(2,0)$ and $(\sqrt{2}, \frac{1}{\sqrt{2}})$. Find the directional derivative of f in the direction of the vector $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ at these two points.

3. Let $f(x,y) = x\tan(xy) + x$. Find the total differential df at the point (0, 1) and use it to approximate $f(0.9, 0.2)$. Do the same thing using the point $(1,0)$.

4. Show that the function $f(r,\theta) = r^2\sin(2\theta)$ is a harmonic function (i.e., show $f_{xx} + f_{yy}$; use that $x = r\cos\theta$ and $y = r\sin\theta$ for polar coordinates).

5. Find the second-order Taylor polynomial of the function $f(x,y) = y\sin x$ at the point $(0,1)$.

5. Consider the two equations:

$$-y^2 + u^2 + v^2 = e^x$$
$$x^2 + u^2 + 2v^2 = y$$

Near what points is it possible to solve locally for u and v as functions of x and y?
Find $\frac{\partial^2 u}{\partial x^2}$.

6. Find the equation of the tangent plane to the implicitly defined surface $x^2 + y^2 = z^2$ at the point $(0, \frac{\pi}{2}, \frac{\pi}{2})$.

7. The temperature on the circular disk $x^2 + y^2 \leq 2$ is given by the function $f(x,y) = x^2 + 2y^2 - 2x$. Find the maximum and minimum of the temperature on the disk.

8. Find all of the critical points and classify them as local maxima, local minima, or saddle points for the function $f(x,y) = x^3 + y^2 - 6xy$.

9. Find the divergence and curl of the vector-valued function
   $\mathbf{F}(x,y,z) = \ln(xz)\,\mathbf{i} + (\arcsin(xy) + xe^z)\mathbf{j} + xy^{\frac{3}{2}}\mathbf{k}$.

10. Answer Exercises 4.6, #1 - #6 in your book.