Name Solutions Permit #

Circle your section number: 1 2 3 4 5 6

There is a second side to the quiz!

1. The equation \(2x - y + 3z = 7\) defines a plane. (a) Find a normal vector to this plane (write it in terms of the standard coordinate vectors \(i, j, \) and \(k\)).

   Note: If \((x_0, y_0, z_0)\) is a point on the plane (here, we could take \((2, 0, 1)\) e.g.),
   then the equation of the plane is \(\mathbf{n} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}\)
   \(\mathbf{n} \cdot (x-2, y, z-1) = 0\) if \(\mathbf{n} = (a, b, c)\), this is \(a(x-2) + b \cdot y + c(z-1) = 0\)

   (b) Compute \(\mathbf{n} \cdot \mathbf{v}\) where \(\mathbf{v} = 3i + 3j - k\). Comparing with the given equation \(\mathbf{n} = (2; 1; 3)\)

   \[
   \mathbf{n} \cdot \mathbf{v} = (2)3 + (-1)(3) + (3)(-1) = 6 - 3 - 3 = 0
   \]

   \[
   \mathbf{n} \cdot \mathbf{v} = 0
   \]

   (c) Is the point \((3, 3, -1)\) on the plane? Write your reason, then also circle your answer in the box below.

   The point \((3, 3, -1)\) does not satisfy the equation

   \(2x-y+3z=7\) \(2 \cdot 3 - 3 + 3(-1) = 6 \neq 7\)

   Circle one: yes \(\bigcirc\) no

2. Consider a scalar function \(f(x, y)\) of two variables.

   (a) Write down the definition of \(\frac{\partial f}{\partial x}(a, b)\). (Hint: The definition must involve a limit.)

   \[
   \frac{\partial f}{\partial x}(a, b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h} = \lim_{x \to a} \frac{f(x, b) - f(a, b)}{x - a}
   \]

   Problem 2 is continued on the back. Note that this is the limit of \(\frac{\Delta f}{\Delta x}\) when only the \(x\)-variable is changing. \(\Delta x\)

   \((y=b \text{ is held constant})\)
(b) If \( f(x, y) = x \arctan(xy) + e^{y^2} \), compute \( \frac{\partial f}{\partial x}(1, 1) \).

(Note: You don't need to use the definition, just compute the partial derivative as usual.)

\[
\frac{\partial f}{\partial x} = 1 \cdot \arctan(xy) + x \frac{\partial}{\partial x}(\arctan(xy)) + \frac{\partial}{\partial x}(e^{y^2}) \\
= \arctan(xy) + x \frac{1}{1 + (xy)^2} \frac{\partial}{\partial x}(xy) + 0 \\
= \arctan(xy) + \frac{xy}{1 + (xy)^2}
\]

Then,

\[
\frac{\partial f}{\partial x}(1, 1) = \arctan(1) + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}
\]