Math 5B: Quiz 2

Name Solutions Perm # _____

Circle your section number: 1 2 3 4 5 6

There is a second side to the quiz!

1. (a) Write a formula for the directional derivative of a function in the direction of the unit vector **u** at a point **p**.

$$D_{\mathbf{u}}f(\mathbf{p}) = \frac{d}{dt} f(\vec{p} + t\vec{u})\Big|_{t=0}$$ OR $\vec{\nabla} f(\vec{p}) \cdot \vec{u}$

(b) At the point $(1,1)$, find the direction in which the function $f(x,y) = x^2 + y$ increases the most rapidly. (Give your answer as a unit vector **u**.)

The maximum rate of increase of $f(x,y)$ at $(1,1)$ occurs in the direction $\vec{\nabla} f(1,1)$.

$\vec{\nabla} f(x,y) = (2x, 1)$

$\vec{\nabla} f(1,1) = (2,1)$ The length of this vector is $\sqrt{2^2 + 1^2} = \sqrt{5}$

So $\vec{u} = \frac{1}{\sqrt{5}}(2,1)$

$$\mathbf{u} = \frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{\jmath}$$

Problem 2 is on the back.

2. Assume we know that a function $g(u, v)$ is a function of the two variables $u(x, y)$ and $v(x, y)$.

(a) Write down the chain rule you would use to find $\frac{\partial g}{\partial y}$.

$$\frac{\partial g}{\partial y} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial y}$$

(b) Consider the function $g(u, v) = u^2 v$, where u and v are the following functions of x and y:

$$u(x, y) = xe^{y^2} \quad \text{and} \quad v(x, y) = x + y$$

Find $\frac{\partial^2 g}{\partial x \partial y}$. Write your final answer in terms of x and y.

First using the chain rule above,

$$\frac{\partial g}{\partial y} = (2uv)(2xye^{y^2}) + (u^2)(1) = 4xyuve^{y^2} + u^2$$

Substituting $u = xe^{y^2}$ and $v = x + y$:

$$\frac{\partial g}{\partial y} = 4xy(xe^{y^2})(x+y)e^{y^2} + (xe^{y^2})^2$$

$$= 4x^3ye^{2y^2} + 4x^2y^2e^{2y^2} + x^2e^{2y^2}$$

Then,

$$\frac{\partial^2 g}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial g}{\partial y}\right) = 12x^2ye^{2y^2} + 8xy^2e^{2y^2} + 2xe^{2y^2}$$

$$\frac{\partial^2 g}{\partial x \partial y} = 2xe^{2y^2}(6xy + 4y^2 + 1)$$