Math 5B: Quiz 2

Name ________________________________  Perm # ______________

Circle your section number: 1 2 3 4 5 6

There is a second side to the quiz!

1. (a) Write a formula for the directional derivative of a function in the direction of the unit vector \( \mathbf{u} \) at a point \( \mathbf{p} \).

\[
D_\mathbf{u} f(\mathbf{p}) = \frac{d}{dt} f(\mathbf{p} + t \mathbf{u}) \bigg|_{t=0} \quad \text{OR} \quad \nabla f(\mathbf{p}) \cdot \mathbf{u}
\]

(b) At the point \((1,1)\), find the direction in which the function \( f(x,y) = x^2 + y \) increases the most rapidly. (Give your answer as a unit vector \( \mathbf{u} \).)

The maximum rate of increase of \( f(x,y) \) at \((1,1)\) occurs in the direction \( \nabla f(1,1) \).

\[
\nabla f(x,y) = (2x, 1)
\]

\[
\nabla f(1,1) = (2, 1) \quad \text{The length of this vector is} \quad \sqrt{2^2 + 1^2} = \sqrt{5}
\]

So \[
\mathbf{u} = \frac{1}{\sqrt{5}} (2, 1)
\]

\[
u = \frac{2}{\sqrt{5}} \hat{i} + \frac{1}{\sqrt{5}} \hat{j}
\]

Problem 2 is on the back.
2. Assume we know that a function \(g(u, v)\) is a function of the two variables \(u(x, y)\) and \(v(x, y)\).

(a) Write down the chain rule you would use to find \(\frac{\partial g}{\partial y}\).

\[
\frac{\partial g}{\partial y} = \frac{\partial g}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial g}{\partial v} \frac{\partial v}{\partial y}
\]

(b) Consider the function \(g(u, v) = u^2v\), where \(u\) and \(v\) are the following functions of \(x\) and \(y\):

\[
u(x, y) = xe^y\quad \text{and} \quad v(x, y) = x + y
\]

Find \(\frac{\partial^2 g}{\partial x \partial y}\). Write your final answer in terms of \(x\) and \(y\).

First using the chain rule above,

\[
\frac{\partial g}{\partial y} = (2uv)(2xy e^{y^2}) + (u^2)(1) = 4xy uv e^{y^2} + u^2
\]

Substituting \(u = xe^y\) and \(v = x + y\):

\[
\frac{\partial g}{\partial y} = 4xy (xe^y)(x+y) e^{y^2} + (xe^y)^2
\]

\[
= 4x^3y e^{2y^2} + 4x^2y^2 e^{2y^2} + x^2 e^{2y^2}
\]

Then,

\[
\frac{\partial^2 g}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial g}{\partial y} \right) = 12x^2y e^{2y^2} + 8xy^2 e^{2y^2} + 2xe^{2y^2}
\]

\[
\frac{\partial^2 g}{\partial x \partial y} = 2xe^{2y^2} (6xy + 4y^2 + 1)
\]