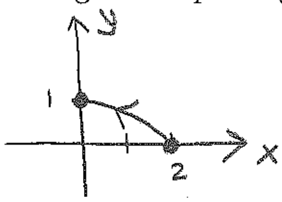
Math 5B: Quiz 3

Name *Solutions* Perm # ____________

Circle your section number: 1 2 3 4 5 6

There is a second side to the quiz!

1. (a) Find a parameterization for the curve that traces out the ellipse $x^2 + 4y^2 = 4$, starting at the point $(2, 0)$ and ending at the point $(0, 1)$.



Notice $(2\cos t)^2 + 4(\sin t)^2$

$= 4[\cos^2 t + \sin^2 t] = 4$

Also $(2\cos(0), \sin(0)) = (2, 0)$

and $(2\cos(\pi/2), \sin(\pi/2)) = (0, 1)$

Note: [There are many other answers! E.g. $\vec{c}_2(t) = (-t, \frac{1}{2}\sqrt{4-t^2})$, $t\in[-2,0]$ works, but using cosine and sine seems natural for circles/ellipses.]

$$\boxed{\mathbf{c}(t) = (2\cos t, \sin t) \quad , t\in[0, \pi/2]}$$

(b) Write the formula you would use to compute the length of the curve in part (a). (Hint: your answer should be in the form of an integral!)

In general, $\ell(\vec{c}(t)) = \int_a^b \|\vec{c}\,'(t)\|\,dt$

For the curve above, $\vec{c}\,'(t) = (-2\sin t, \cos t)$

and $\|\vec{c}\,'(t)\| = \sqrt{4\sin^2 t + \cos^2 t} = \sqrt{3\sin^2 t + 1}$

$$\boxed{\text{length of } \mathbf{c}(t) = \int_0^{\pi/2} \sqrt{3\sin^2 t + 1}\,dt}$$

Problem 2 is on the back.

2. If you know that the velocity of a particle is given by the formula $\mathbf{v}(t) = 2e^t\mathbf{i} + t\mathbf{j}$, and if you know the position at time $t = 1$ is $\mathbf{c}(1) = (2e, 1, 2)$, find a formula for $\mathbf{c}(t)$.

Integrating the velocity to find $\vec{c}(t)$

$$\vec{c}(t) = (2e^t + c_1)\hat{\imath} + \left(\frac{t^2}{2} + c_2\right)\hat{\jmath} + c_3\hat{k}$$

We want

$$\vec{c}(1) = 2e\hat{\imath} + \hat{\jmath} + 2\hat{k}, \text{ so } c_1 = 0$$

$$c_2 = \frac{1}{2}$$

$$c_3 = 2.$$

$$\boxed{\mathbf{c}(t) = 2e^t\hat{\imath} + \left(\frac{t^2}{2} + \frac{1}{2}\right)\hat{\jmath} + 2\hat{k} \quad \underline{\text{OR}} \quad \left(2e^t, \frac{t^2}{2} + \frac{1}{2}, 2\right)}$$

(b) Write a parametric equation for the tangent line to $\mathbf{c}(t)$ at time $t = 1$.

We Know $\vec{c}(1) = (2e, 1, 2)$ is a point on the line and the tangent vector is $\vec{v}(1) = 2e\hat{\imath} + \hat{\jmath} = (2e, 1, 0)$

$$\vec{l}(t) = (2e, 1, 2) + t(2e, 1, 0)$$

$$\boxed{\mathbf{l}(t) = (2e + (2e)t,\ 1 + t,\ 2)}$$