Math 5B: Final Review Problems

Here are some extra review problems for the final. You should also look over your homework, old tests and quizzes, and the midterm review problems!

Some useful formulas:

The area of the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ is πab.

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

1. Review limits, partial derivatives (including the chain rule and implicit differentiation), tangent lines, and tangent planes.

2. Find the global minimum and the global maximum of the function $f(x,y) = \ln(x^2+y^2) + x$ inside the circle of radius 1 and centered at the point $(-2,0)$.

3. Find all local minima, local maxima, and saddle points of the function $f(x,y) = e^{x^2+y^2} - 2y^2$.

4. Let $\mathbf{v} = (6x^2y + x - 1)\mathbf{i} + (z\sec^2(y) + 2x^3)\mathbf{j} + \tan(y)\mathbf{k}$. Verify that curl $\mathbf{v} = 0$. Find all functions f such that $\mathbf{v} = \nabla f$.

5. Evaluate $\frac{\partial}{\partial s}\int_0^t \frac{e^{-s^2(u^2+1)}}{u^2+1}\,du$ and $\frac{\partial}{\partial t}\int_0^t \frac{e^{-s^2(u^2+1)}}{u^2+1}\,du$.

6. Consider the change of variables

$$\begin{aligned} u &= xy \\ v &= x^2 - y^2 \end{aligned}$$

What region is the square in the xy-plane $0 \le x \le 1$ and $0 \le y \le 1$ mapped to in the uv-plane? (Hint: Where is each line on the boundary of the square – $x = 0$, $x = 1$, $y = 0$, and $y = 1$ – mapped to in the uv-plane?) Evaluate the following integral using this change of variables:

$$\iint_{R_{xy}} xy(x^2+y^2)\sqrt[3]{x^2 + x^2y^2 - y^2}\,dx\,dy$$

where R_{xy} is the square $0 \le x \le 1$ and $0 \le y \le 1$.

7. Evaluate

$$\iint_R \frac{y}{x} e^{x^2+y^2}\, dx\, dy,$$

where R is the region bounded by the lines $y = x$, $y = 0$ and the half-circle $x = \sqrt{1-y^2}$.

8. (a) Evaluate $\displaystyle\int_0^1 \int_1^y \frac{1}{1+y^2}\, dx\, dy$.

(b) Evaluate $\displaystyle\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$. (Hint: There's no way to find an antiderivative for $\frac{\sin x}{x}$!)

9. Find the volume of the region inside the cylinder $x^2 + y^2 = 1$ and the sphere of radius 2 centered at the origin $(0, 0, 0)$.

10. Find the area of a region bounded by the lines $x = -\frac{1}{\sqrt{2}}$, $x = \frac{1}{\sqrt{2}}$, and the circle of radius 1 centered at $(0, 0)$.

11. Set up the integral (but do not evaluate!!) that will give you the length of the curve given by the following parametric equations: For $0 \le t \le \pi$,

$$\begin{aligned} x &= 3\cos(t) + 5 \\ y &= \sin(3t) \end{aligned}$$

12. Verify Green's theorem by computing the following integral in two different ways (once by doing a line integral and once by doing a double integral):

$$\oint_C ye^x dx + xdy$$

where C is the triangle with vertices (0,0), (3,0), and (1,2).

13. Evaluate the following integrals:

(a) $\oint_C xy^6 dx + (3x^2y^5 + 6x)\, dy$ where C is the ellipse $x^2 + 4y^2 = 4$.

(b) $\int_{(1,0)\,C}^{(2,2)} (2xe^y - 1)\, dx + x^2e^y\, dy$ where C is the parabola $y = 2(x-1)^2$.

(c) $\int_{(0,1)\,C}^{(2,e^2)} x^2y\, dx + xy^2\, dy$ where C is the curve $y = e^x$.

(d) $\oint_C x^2\, ds$ where C is the circle of radius 2 centered at $(1, 0)$.