Math 5B: Midterm Review Problems

1. Find the value of the limit if it exists. Otherwise, prove the limit does not exist.
   (a) $\lim_{(x,y)\to(1,0)} e^{-\frac{x}{y^2}}$
   (b) $\lim_{(x,y)\to(1,0)} \dfrac{xy(y-x+1)}{(x-1)^2+y^2}$.

2. Let $f(x,y) = x^2 + 4y^2$. Draw some of the level curves of the surface $z = f(x,y)$ (e.g., draw the curves that correspond to $z = 0, z = 1, z = 4, z = 9$). Find the gradient of f, ∇f. Draw the gradient vectors of f at the points $(2,0)$ and $(\sqrt{2}, \frac{1}{\sqrt{2}})$. Find the directional derivative of f in the direction $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ (i.e., compute $\nabla_{\mathbf{v}} f$).

3. Let $z = f(x,y) = x\tan(xy) + x$. Find the total differential dz. Use the total differential at the point $(1,0)$ to approximate $f(0.9, 0.2)$.

4. Show that the function f, given in polar coordinates as $f(r,\theta) = r^2\sin(2\theta)$, is a harmonic function.

5. Assume u and v are implicit functions of x and y defined by

$$\begin{aligned} -y^2 + u^2 + v^2 &= e^x \\ x^2 + u^2 + 2v^2 &= y \end{aligned}$$

Find $\dfrac{\partial^2 u}{\partial x^2}$.

6. Consider the following parametric equations for a curve in space

$$\begin{aligned} x(t) &= t\cos(t) \\ y(t) &= t\sin(t) \\ z(t) &= t \end{aligned}$$

   (a) Show the curve lies on the surface $x^2 + y^2 = z^2$.
   (b) Find the velocity vector $\mathbf{v}(t)$ for the curve at the point $(0, \frac{\pi}{2}, \frac{\pi}{2})$.
   (c) Write the parametric equations of the tangent line to the curve at the point $(0, \frac{\pi}{2}, \frac{\pi}{2})$.

7. Find the equation of the tangent plane to the surface $x^2 + y^2 = z^2$ at the point $(0, \frac{\pi}{2}, \frac{\pi}{2})$.

8. The temperature on a circular disc $x^2 + y^2 \leq 2$ is given by the function $f(x, y) = x^2 + 2y^2 - 2x$. Find the maximum and minimum values of the temperature on the disc.

9. Find all of the critical points and classify them as local maxima, local minima, or saddle points for the function $f(x, y) = x^3 + y^2 - 6xy$.