Math 5B: Answers to Midterm Review Problems

1. (a) $\lim_{(x,y)\to(1,0)} e^{-\frac{x}{y^2}} = 0$

(b) $\lim_{(x,y)\to(1,0)} \frac{xy(y-x+1)}{(x-1)^2+y^2}$ does not exist
Along the line $x = 1$, the limit equals 1, but along the line $y = 0$, the limit equals 0.

2. The level curves of the surface are the ellipses $x^2 + 4y^2 = c$ for $c \geq 0$.
$\nabla f = 2x\ \mathbf{i} + 8y\ \mathbf{j}$, so $\nabla f(2,0) = 4\ \mathbf{i}$ and $\nabla f(\sqrt{2}, \frac{1}{\sqrt{2}}) = 2\sqrt{2}\ \mathbf{i} + 4\sqrt{2}\ \mathbf{j}$
$\nabla_{\mathbf{v}} f(x,y) = \dfrac{2x+16y}{\sqrt{5}}$

3. $dz = (\tan(xy) + yx\ \sec^2(xy) + 1)\, dx + (x^2\ \sec^2(xy))dy$
At $(1,0)$, $dz = dx + dy$; therefore, $f(0.9, 0.2) \approx f(1,0) + \Delta x + \Delta y = 1.1$.

4. $f_{rr} = 2\sin(2\theta)$; $\dfrac{f_r}{r} = 2\sin(2\theta)$; $\dfrac{f_{\theta\theta}}{r^2} = -4\sin(2\theta)$. Then, $\nabla^2 f = f_{rr} + \dfrac{f_r}{r} + \dfrac{f_{\theta\theta}}{r^2} = 0$

5. $\dfrac{\partial u}{\partial x} = \dfrac{e^x + x}{u}$ and $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{e^x+1}{u} - \dfrac{(e^x+x)^2}{u^3}$

6. (a) $(t\cos(t))^2 + (t\sin(t))^2 = t^2$
(b) At the point $(0, \frac{\pi}{2}, \frac{\pi}{2})$, $t = \frac{\pi}{2}$, and $\mathbf{v}(\frac{\pi}{2}) = -\frac{\pi}{2}\mathbf{i} + \mathbf{j} + \mathbf{k}$
(c) $x(s) = -\frac{\pi}{2}s$
$y(s) = s + \frac{\pi}{2}$
$z(s) = s + \frac{\pi}{2}$

7. $y - z = 0$

8. The minimum value of f is -1 (this occurs at the critical point (1,0)).
The maximum value of f is 5 (this occurs at the boundary points (-1, 1) and (-1,-1)).

9. There are two critical points of f: $a = (0,0)$ and $b = (6,18)$.
The matrix of second derivatives of f is $\text{Hess}(f)(x,y) = \begin{bmatrix} 6x & -6 \\ -6 & 2 \end{bmatrix}$.
At a, $\det[\text{Hess}(f)(0,0)] = -36 < 0$, so a is a saddle point.
At b, $\det[\text{Hess}(f)(6,18)] = 36 > 0$, and since $f_{xx}(6,18) = 36 > 0$, b is a local minimum.